S. KOMENDA, J. KLEMENTA AND J. KRÁTOŠKA

PROPORTION OF BODY DIMENSIONS IN ADULT POPULATION

Mass production of goods in whose parameters body dimensions of their future consumers have to be respected is one of typical features in the contemporary society. These consumers are anonymous from the point of view of the producer; they are elements of a population for him, whose properties can be described in a statistical way (Komenda and Krátoška 1968, 1974a, 1977; Kuršakova et al. 1974).

Mass production is conditioned by determination of a set of type figures able to represent with an accuracy high enough the population under consideration. Such a set of type figures is denoted as antropometrical standard of the population (Kuršakova et al. 1974). In the production goods are designed and constructed just for these type figures corresponding to the anthropometrical standard.

In determination of an anthropometrical standard not all body dimensions important from the standpoint of the goods designer are considered. As known generally in anthropology, body dimensions correlate mutually so that knowledge of the values of some principal body dimensions among those under consideration gives an opportunity to estimate with sufficient accuracy the values of remaining body dimensions (Klementa et al. 1973). Methods of multiple regression and correlation analysis are used in it under the assumption that the body dimensions under consideration have multivariate normal probability distribution in the population of future consumers (Kuršakova et al. 1974).

In the conception of "sufficient accuracy" there is very narrow correspondence to the way by means of which type figures have been determined: provided that the values of principal body dimensions are known possible values of the remaining body dimensions are expected with the probability sufficiently high in an area which can be represented by just one type figure.

Principles introduced above play an important part in many branches of modern industry such as garment-, underwear- and footwear production as well as that of tool and equipment designing (Damon et al. 1966; Kuršakova et al. 1974; Steigl et al. 1976).

In the design and construction system dependence of body dimensions taken into account on the principal body dimensions is expressed sometimes in the form of the proportion of an inferior body dimension to the principal one (Fetter et al. 1967; Martin and Saller 1959). That was the reason why we decided to study statistical properties of proportions theoretically (Komenda and Krátoška 1974b; Krátoška et al. 1974).

Main results of our study consist in the possibility to compute approximately expectation of proportions and the variance of them in the whole population of subjects as well as in the subpopulations suitably delimited in it (Komenda and Klementa 1978).

The results derived theoretically have been applied in anthropological data obtained by the research workers in the Garment Research Institute (VÚO) in Prostějov (Head of the Department: Dipl. techn. Č. Růžička) in 1967, as a part of an extensive research programme. 16 body dimensions have been selected for the purpose of this communication, as given in the following list. 1,392 men and 1,373 women were measured, in the age range from 18 to 60 years.

Numerical calculations have been carried out in the computer C 8206 in Medical Faculty of the Palacký University in Olomouc. Computation program has been elaborated by Mrs. A. Tománková, data preparation and a part of graphical documantation were due Mrs. J. Jelínková. Authors thank them in this place for their kind collaboration.

LIST OF BODY DIMENSIONS

X... Stature (1)
Y... Chest Circumference (16)
W... Neck Circumference (13)
Waist Circumference (18)
Buttock Circumference (19)
Thigh Circumference (21)
Upper Arm Circumference (28)
Body Mass (59)
Waist Height (7)
Knee Height (9)
Height of the 7 th Neck Vertebra (10)
Gluteal Furrow Height (12)
Hip Height*)
Shoulder Breadth (53)
Hip Breadth (56)
Chest Depth (58)

Numbers in the brackets correspond to those given in Kuršakova et al. (1974), where also the definitions of body dimensions can be found.

PROBABILITY DISTRIBUTION OF PROPORTION IN GENERAL CASE

Let us consider pair of body dimensions (X, W) under the assumption that conditional probability distribution of the dimension W providing that $X = x$ is $f_1(w/x)$ which can be with a sufficient approximation taken as normal. This pair (X, W) will be considered asymmetrically in the following sense: dimension W is taken as inferior one while X is taken as principal dimension. By this it may be expressed that the knowledge of the value of the dimension X is to be used in estimation of the value of the dimension W.

We may ask now which percentage of X is W and if this percentage is depending on the value of X. Especially, this percentage is to be determined in certain subpopulations of the dimension X suitably chosen. Provided that X and W are considered as random variables, the percentage is random variable, too, and statistical properties of it can be studied: its probability distribution or at least its expectation and variance. As a special case of this problem

*) *Vertical distance from floor to the horizontal plane in which buttock circumference is measured.*

FIG. 1.

Theoretical frequency distribution of the basic body dimension.

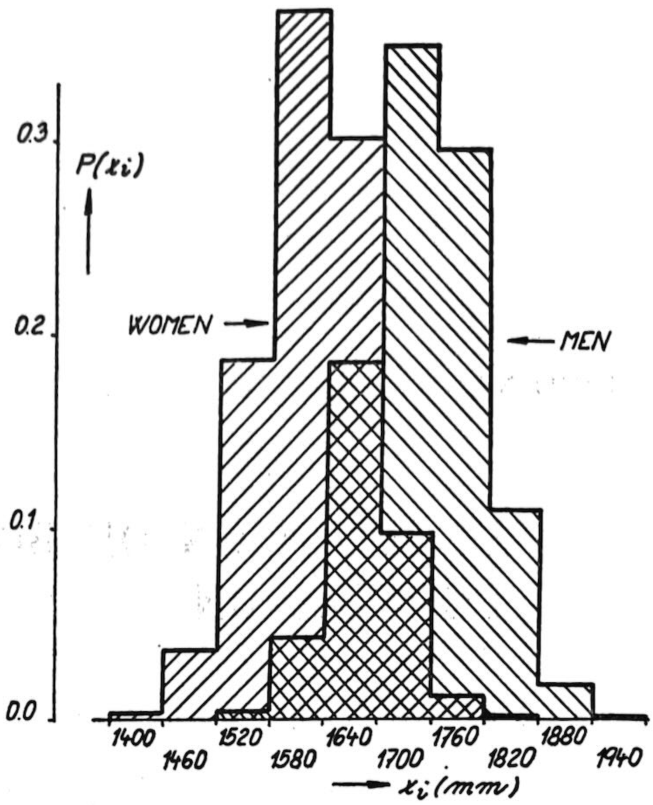

FIG. 1—1

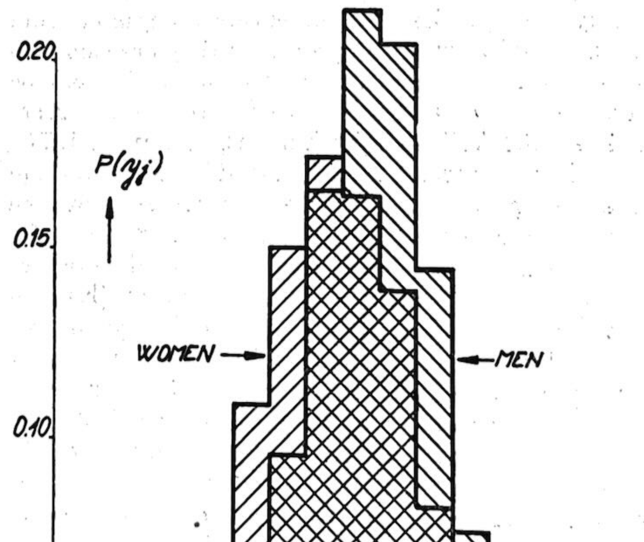

FIG. 1—2

FIG. 2.

Mean proportion $E(P \mid x_i)$ or $E(P \mid y_j)$ and the range of variation $\pm \sqrt{D(P \mid x_i)}$ or $\pm \sqrt{D(P \mid y_j)}$, as a function of the basic body dimension X or Y.

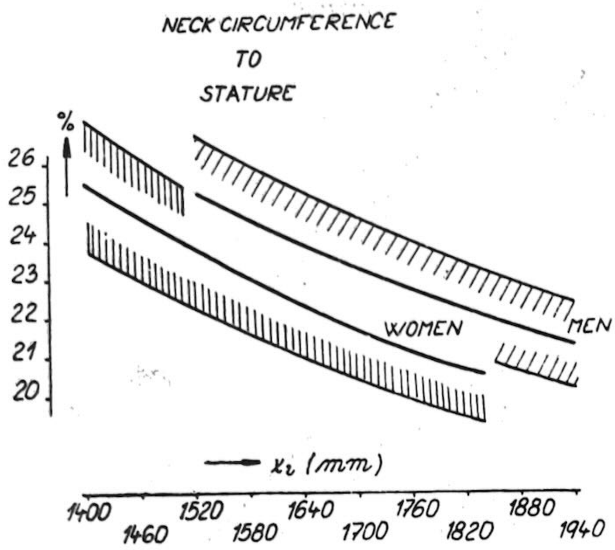

FIG. 2—1

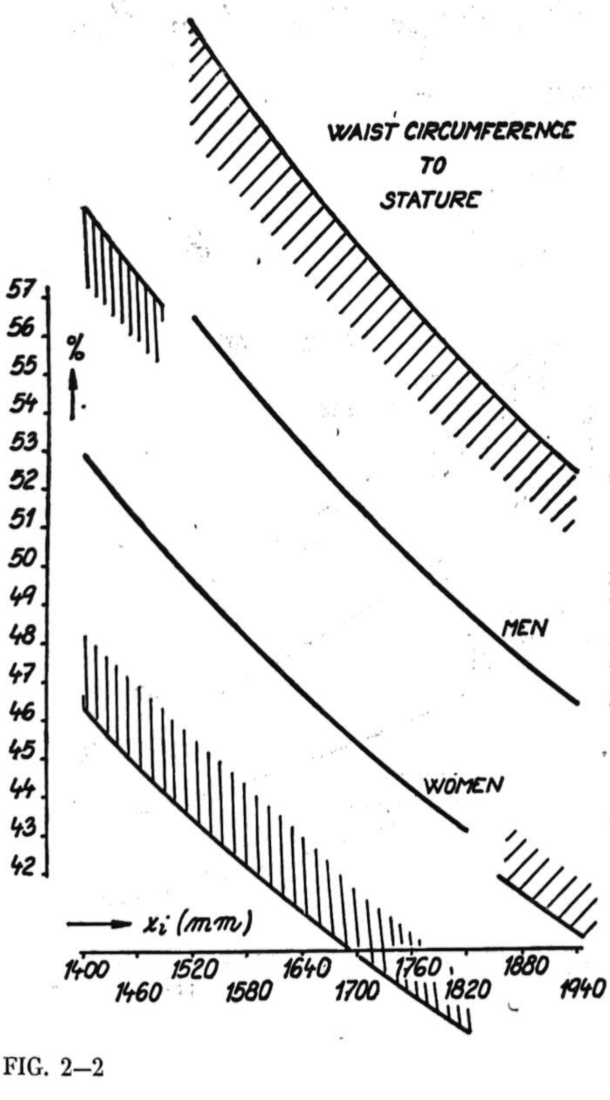

FIG. 2—2

proportion of body dimension W to the dimension X in the whole population of adult subjects can be studied (Bunak 1941; Fieller 1932; Fisz 1958; Gnedenko 1976; Kendall and Stuart 1962).

Proportion of body dimension W to the principal body dimension X (in the hypothetical subpopulation $X = x$) is defined as follows

$$p = \frac{W}{x} 100\ (\%). \tag{1}$$

As far as the value x is considered as constant, probability density $g(p)$ of the random variable P is easy to derive. It holds namely

$$g(p \mid x)\, \mathrm{d}p = f_1(w \mid x)\, \mathrm{d}w \tag{2}$$

where

$$w = w(p) = \frac{px}{100}$$

or

$$g(p \mid x) = f_1(w \mid x) \frac{x}{100} \tag{2a}$$

where it is necessary to substitute from the foregoing expression for w.

In reality, no non-empty subpopulation can be delimited by the condition $X = x$, if there is not an interval of x - values considered by it. Hence, the subpopulations are defined in such a way so that each of them includes all subjects from the population under consideration whose values of the dimension X are in the interval x_i determined by the inequalities

$$x_i - \Delta_X \leqq X < x_i + \Delta_X \quad (i = 1, 2, \ldots, q) \tag{3}$$

By the symbols x_i intervals of the dimension X of the form (3) are denoted and simultaneously subpopulations of subjects belonging into these intervals. The constant Δ_X is chosen so that the interval is suitable for the purpose the proportions are appointed. In our case where stature X and chest circumference (the symbol Y is used for it) are principal body dimensions, subpopulations (3) were established by the choice $\Delta_X = 30$ mm and $\Delta_Y = 20$ mm.

In the subpopulation x_i probability distribution of the proportion P is given by the probability density

$$g(p \mid x_i) = \frac{1}{P(x_i)} \int_{x_i - \Delta_X}^{x_i + \Delta_X} \frac{x}{100} f_1\left(\frac{px}{100}\right) f(x)\, \mathrm{d}x \tag{4}$$

where

$$P(x_i) = \int_{x_i - \Delta_X}^{x_i + \Delta_X} f(x)\, \mathrm{d}x$$

is the probability of occurrence of subjects with their x-values of X belonging into the interval x_i, in the considered population.

Provided that the whole population is considered instead of partial subpopulations (4) becomes

$$g(p) = \int \frac{x}{100} f_1\left(\frac{px}{100}\right) f(x)\, \mathrm{d}x \tag{5}$$

By this probability distribution of the proportion P in the whole population of subjects under consideration is given.

Thus it is obvious that probability distribution of the proportion P which expresses the ratio of body dimension W to the principal dimension X can be derived by means of the probability distribution $f(x)$ of X and of the conditional probability distribution $f(w \mid x)$ of W provided that $X = x$ is given. Knowledge of these initial distributions is necessary if the probability distribution of the proportion P should be derived, in the whole population as well as in the subpopulations x_i suitably chosen.

APPROXIMATE DERIVATION OF EXPECTATIONS AND VARIANCES OF THE PROPORTION P IN THE PARTIAL SUBPOPULATIONS X_i AND IN THE WHOLE POPULATION

Let us suppose conditional probability distribution of body dimension W in the subpopulation $X = x$ to be normal with the probability density

$$f_1(w / x) = \frac{1}{\sqrt{2\pi\sigma_{W.X}}} \exp\left\{-\frac{1}{2}\frac{(w-\mu_{W.x})^2}{\sigma_{W.X}}\right\} \tag{6}$$

$$(-\infty < w < \infty)$$

where

$$\mu_{W.x} = \mu_W + \beta_{W.X}(x - \mu_X)$$
$$\sigma_{W.X} = \sigma_{WW} - \sigma_{WX}^2/\sigma_{XX}$$
$$\beta_{W.X} = \sigma_{WX}/\sigma_{XX}$$

are conditional expectation of the dimension W in the subpopulation $X = x$, residual variance of the dimension W and regression coefficient of W on the principal dimension X. All characteristics which occur in the right-hand side of the last three formulae are estimable immediately from anthropometrical data.

Provided that x is taken as constant, probability distribution of the proportion (1) is normal, i.e. $g(p / x)$ is Gaussian probability density. Distribution parameters of it are as follows:

Expectation (mean value)

$$E(P / x) = \int p\, g(p / x)\, dp$$
$$= \frac{100}{x}\mu_{W.x} \tag{7}$$
$$= 100 \left[\beta_{W.X} + \frac{\mu_W - \beta_{W.X}\mu_X}{x}\right]$$

Variance

$$D(P / x) = \int [p - E(P / x)]^2\, g(p / x)\, dp$$
$$= \left(\frac{100}{x}\right)^2 \sigma_{W.X} \tag{8}$$

As can be seen immediately from (7), expected proportion of the dimension W to the principal dimension X is the hyperbolic function of x. Besides this important part of the regression coefficient $\beta_{W.X}$ is obvious: expected proportion equals just 100-mul-

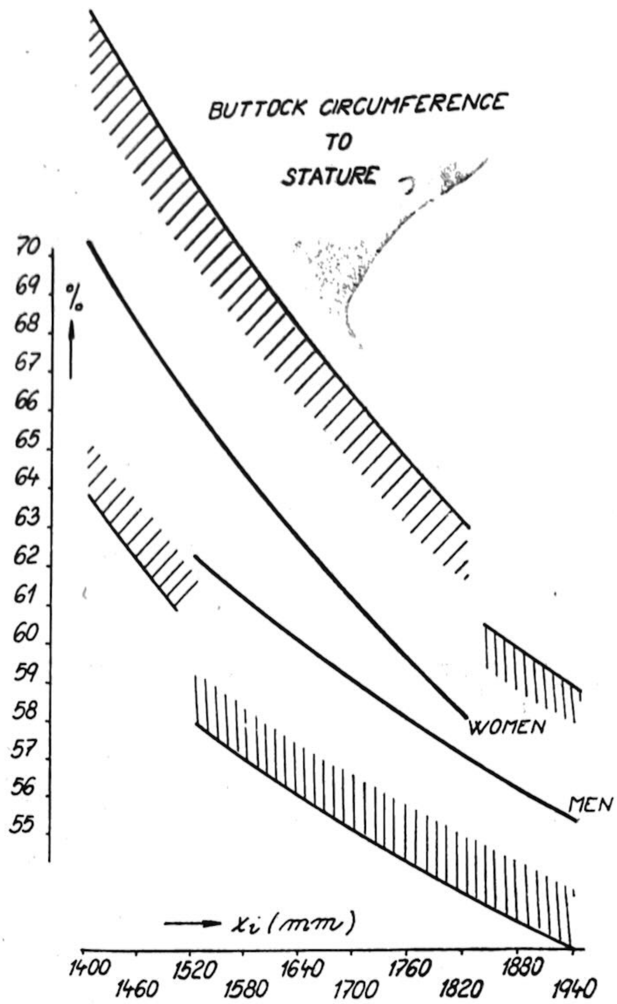

FIG. 2—3

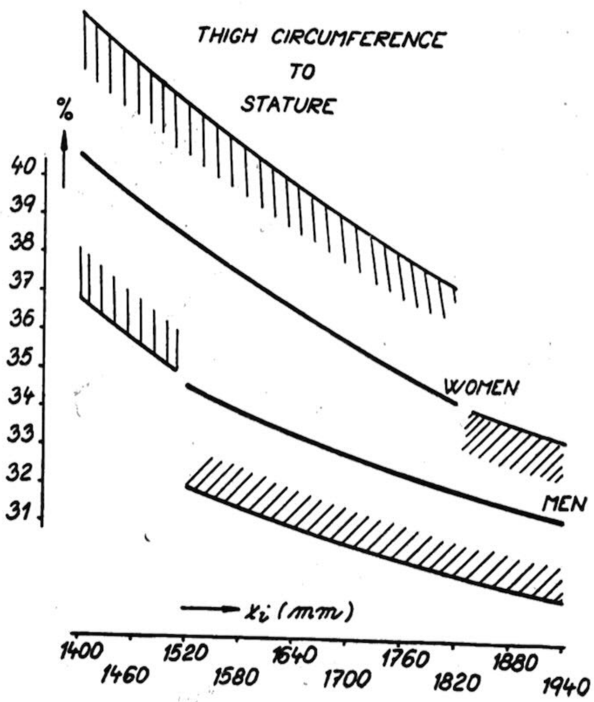

FIG. 2—4

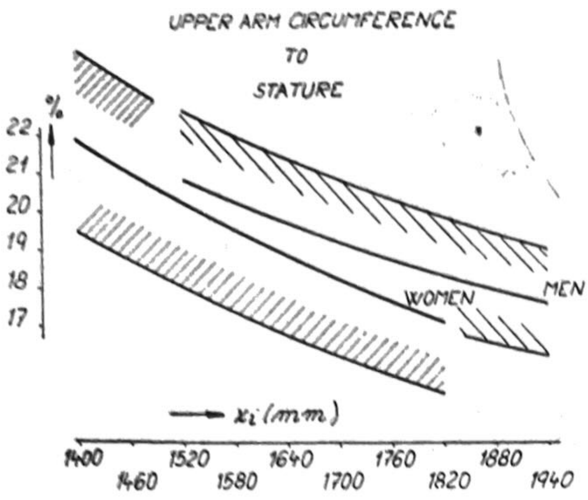

FIG. 2–5

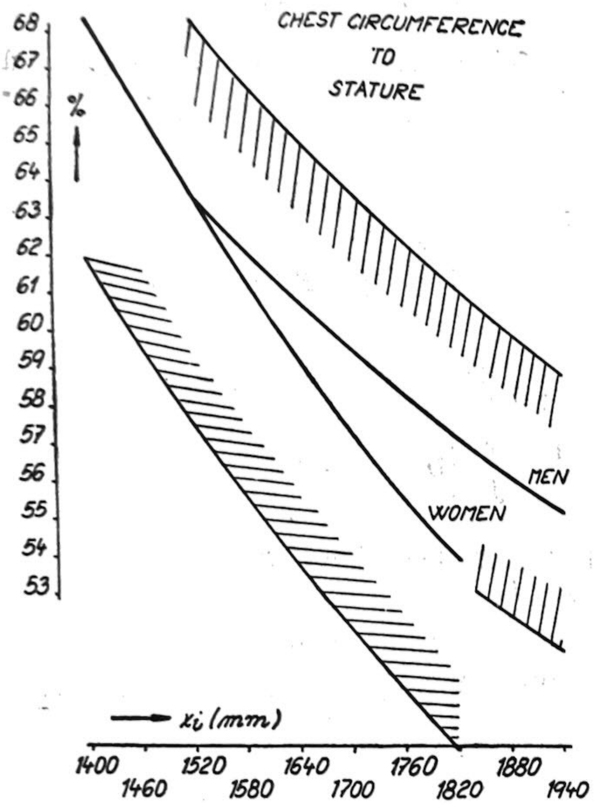

FIG. 2–6

tiple of the regression coefficient if the expression $\mu_W - \beta_{W.X}\mu_X$ equals zero, i.e. if $\beta_{W.X} = \mu_W/\mu_X$. As it is known from regression analysis this is the case when the regression line goes through the origin of the coordinate system. Hence, expected proportion of the dimension W to the dimension X is constant in the whole range of x-values just in this case $\beta_{W.X} = \mu_W/\mu_X$ (coefficient of regression equals the ratio μ_W/μ_X of the expectation of both body dimensions followed up).

In the subpopulation x_i probability distribution of the proportion P is given by the probability density

$$g(p \mid x_i) = \frac{1}{P(x_i)} \int_{x_i - \Delta_X}^{x_i + \Delta_X} g(p \mid x) f(x) \, \mathrm{d}x \qquad (9)$$

and — if the distribution of the dimension X can be approximated in the subpopulation x by the uniform density —

$$\frac{f(x)}{P(x_i)} \sim \frac{1}{2\Delta_X}, \; (x_i - \Delta_X \leqq x < x_i + \Delta_X) \qquad (10)$$

the following is obtained approximately for (9)

$$g(p \mid x_i) = \frac{1}{2\Delta_X} \int_{x_i - \Delta_X}^{x_i + \Delta_X} g(p \mid x) \, \mathrm{d}x \qquad (11)$$

accuracy of which is sufficient if the values of Δ_X are not too high.

Distribution (9) makes it possible to derive approximative expressions of expectation and of variance of the proportion in the subpopulation x_i $(i = 1, 2, \ldots, q)$. It holds namely

$$E(P \mid x_i) = \int p \, g(p \mid x_i) \, \mathrm{d}p$$

Substitution of (11) for $g(p \mid x_i)$ and interchanging of order of integration yields

$$E(P \mid x_i) = \frac{1}{2\Delta_X} \int_{x_i - \Delta_X}^{x_i + \Delta_X} E(P \mid x) \, \mathrm{d}x$$

where there is necessary to set from (7) for $E(P \mid x)$. Integration yields finally

$$E(P \mid x_i) =$$
$$= 100 \left\{ \beta_{W.X} + (\mu_W - \beta_{W.X}\mu_X) \frac{1}{2\Delta_X} \ln \frac{x_i + \Delta_X}{x_i - \Delta_X} \right\}$$

(ln = logarithmus naturalis) (12)

Formula (12) makes it possible to find expectation $E(P \mid x_i)$ in the subpopulation x_i by means of the known coefficient of regression, expectations of both considered body dimensions X and W and of the auxiliar function $\ln[(x_i + \Delta_X)/(x_i - \Delta_X)]$.

By means of approximation (10) also the expression for the expected proportion $E(P)$ in the whole population can be found. It holds namely

$$E(P) = \int p g(p) \, \mathrm{d}p$$

where it is necessary to substitute for $g(p)$ from the relation

$$g(p) = \int g(p \mid x) f(x) \, \mathrm{d}x$$

In this manner, interchanging of both integrations yields

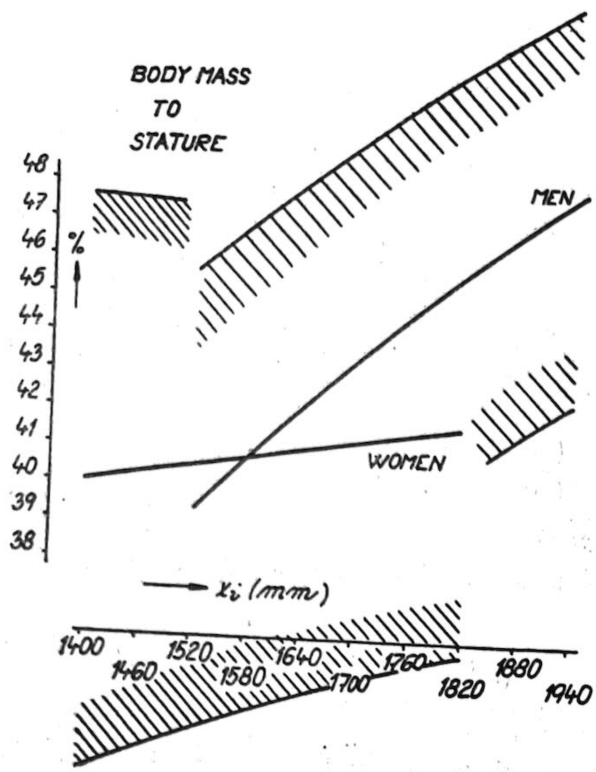

FIG. 2–7

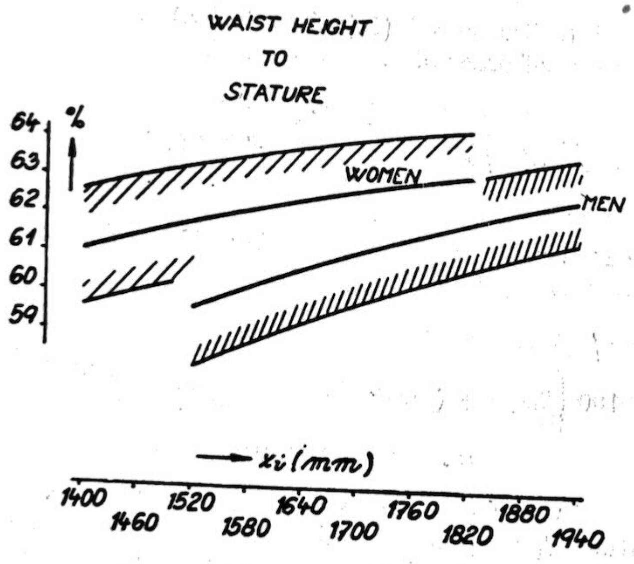

FIG. 2–8

$$E(P) = \int [\int p\, g(p \mid x)\, \mathrm{d}p]\, f(x)\, \mathrm{d}x$$
$$= \int E(P \mid x)\, f(x)\, \mathrm{d}x$$

Substitution from (7) for $E(P \mid x)$ is carried out and the range of integration of the variable x is divided exhaustively into a system of disjoint intervals x_i, $i = 1, 2, \dots, q$; in each of them approximation (10) is used through which $f(x)$ is replaced by the uniform division of the probability mass

$$P(x_i) = \int_{x_i - \Delta x}^{x_i + \Delta x} f(x)\, \mathrm{d}x$$

This yields

$$E(P) = \frac{100}{2\Delta x} \sum_{i=1}^{q} P(x_i) \int_{x_i - \Delta x}^{x_i + \Delta x} \left[\beta w.x + \frac{\mu w - \beta w.x \mu x}{x}\right] \mathrm{d}x$$

and after integration approximative expression for the expected proportion $E(P)$ is obtained in the form

$$E(P) = 100 \left\{\beta w.x + \frac{\mu w - \beta w.x \mu x}{2\Delta x} \sum_{i=1}^{q} P(x_i) \ln \frac{x_i + \Delta x}{x_i - \Delta x}\right\} \tag{13}$$

Similarly, approximative expressions for the variance of proportion can be derived. It holds

$$D(P \mid x_i) = \int [p - E(P \mid x_i)]^2\, g(p \mid x_i)\, \mathrm{d}p$$

After substitution from (11) for $g(p \mid x_i)$ and an interchange of integrations approximative expression for the variance of the proportion is obtained step by step

$$D(P \mid x_i) =$$
$$= \frac{1}{2\Delta x} \int_{x_i - \Delta x}^{x_i + \Delta x} \left\{ \int [p - E(P \mid x)]^2\, g(p \mid x)\, \mathrm{d}p + \int [E(P \mid x) - E(P \mid x_i)]^2\, g(p \mid x)\, \mathrm{d}p \right\} \mathrm{d}x$$
$$= \frac{1}{2\Delta x} \int_{x_i - \Delta x}^{x_i + \Delta x} \left\{ D(P \mid x) + [E(P \mid x) - E(P \mid x_i)]^2 \right\} \mathrm{d}x$$

and after substitution from (7) and (8) for $E(P \mid x)$ and $D(P \mid x)$ and from (12) for $E(P \mid x_i)$ we obtain

$$D(P \mid x_i) = \frac{1}{2\Delta x} \int_{x_i - \Delta x}^{x_i + \Delta x} \left\{ \left(\frac{100}{x}\right)^2 \sigma w.x + 100^2 (\mu w - \beta w.x \mu x)^2 \left[\frac{1}{x} - \frac{1}{2\Delta x} \ln \frac{x_i + \Delta x}{x_i - \Delta x}\right]^2 \right\} \mathrm{d}x$$

The integration yields

$$D(P \mid x_i) = \frac{100^2}{2\Delta x} \left\{ \sigma w.x \frac{2\Delta x}{(x_i + \Delta x)(x_i - \Delta x)} + (\mu w - \beta w.x \mu x)^2 . \left[\frac{2\Delta x}{(x_i + \Delta x)(x_i - \Delta x)} - \frac{2}{2\Delta x} \left(\ln \frac{x_i + \Delta x}{x_i - \Delta x}\right)^2 + \frac{2\Delta x}{(2\Delta x)^2} \left(\ln \frac{x_i + \Delta x}{x_i - \Delta x}\right)^2\right] \right\}$$

and after some adjustment we obtain finally

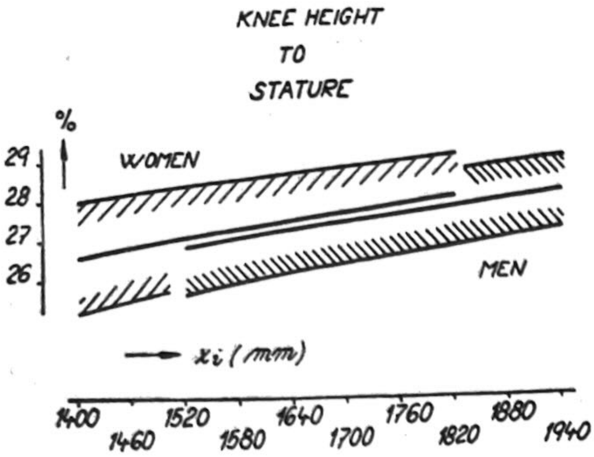

FIG. 2—9

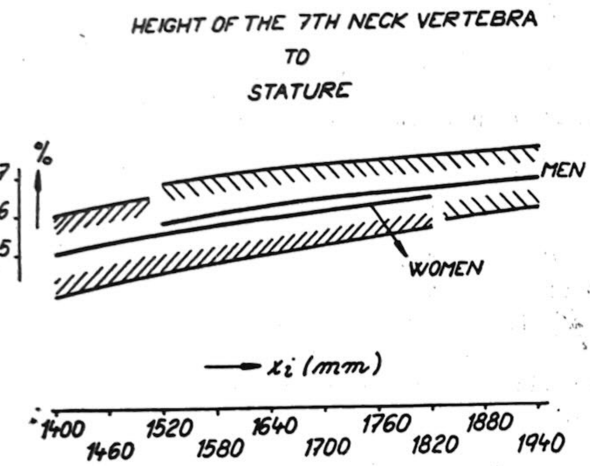

FIG. 2—10

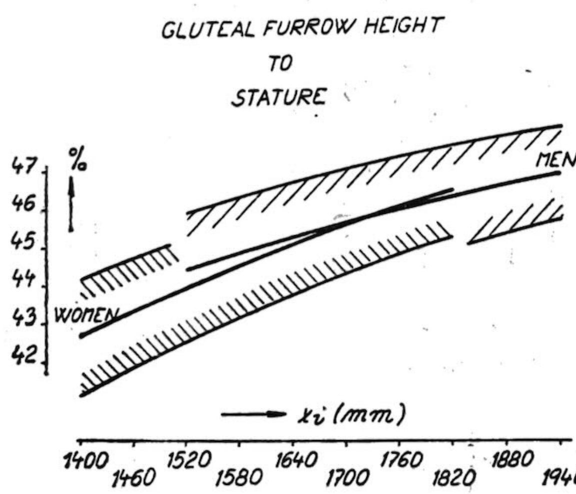

FIG. 2—11

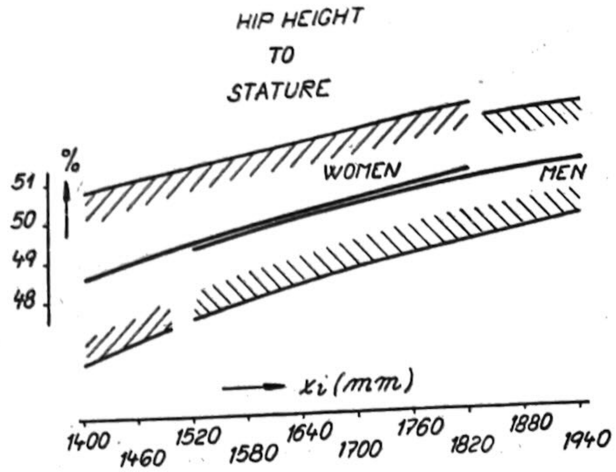

FIG. 2—12

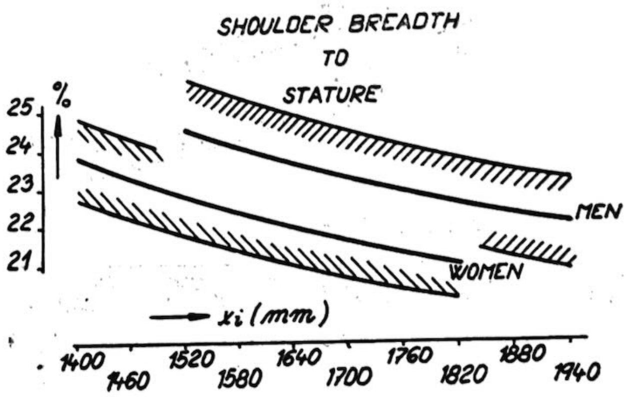

FIG. 2—13

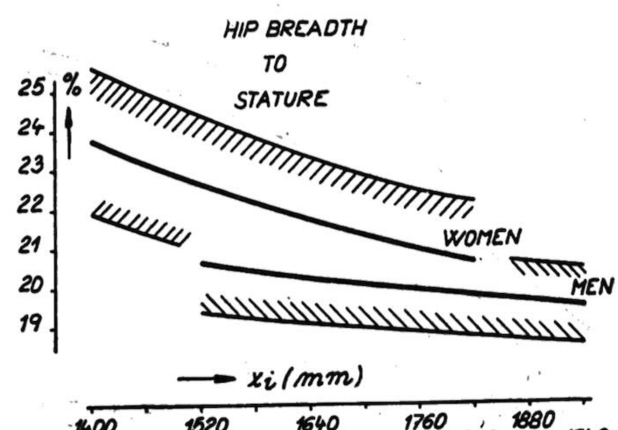

FIG. 2—14

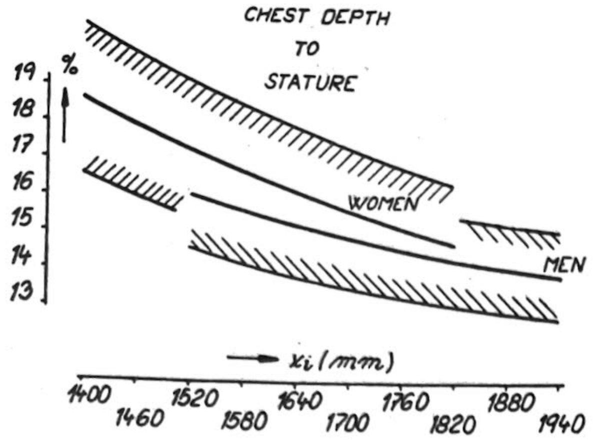

FIG. 2—15

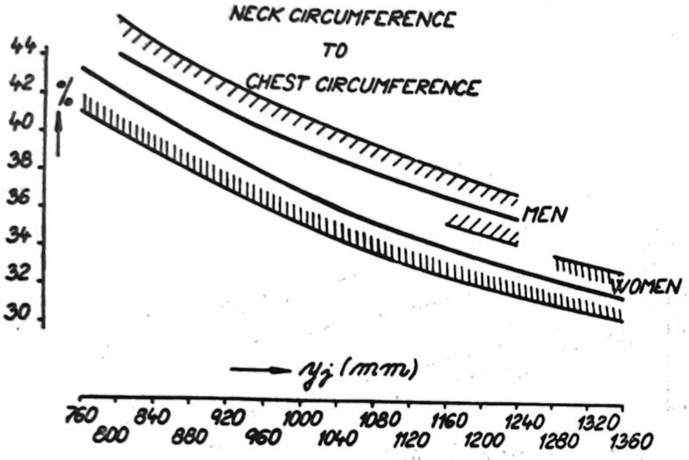

FIG. 2—16

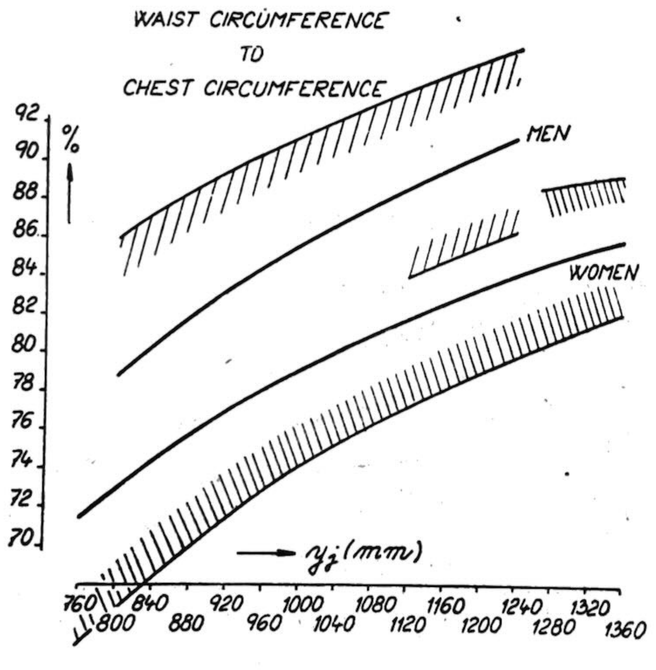

FIG. 2—17

$$D(P \mid x_i) = 100^2 \left\{ \sigma_{W.X} \frac{1}{(x_i + \Delta_X)(x_i - \Delta_X)} + \right.$$
$$+ (\mu_W - \beta_{W.X} \Delta_X)^2 . \qquad (14)$$
$$\left. \left[\frac{1}{(x_i + \Delta_X)(x_i - \Delta_X)} - \frac{1}{(2\Delta_X)^2} \left(\ln \frac{x_i + \Delta_X}{x_i - \Delta_X} \right)^2 \right] \right\}$$

As can be seen, variance of the proportion P in the subpopulation x_i can be derived by means of the known residual variance $\sigma_{W.X}$ of the dimension W provided that X is known, by means of the expectations μ_W and μ_X, regression coefficient $\beta_{W.X}$ and auxiliar functions $\ln [x_i + \Delta_X) / (x_i - \Delta_X)]$ and $1/(x_i + \Delta_X)(x_i - \Delta_X)$.

Further on, approximative formula for the variance of proportion in the whole population is derived. This variance as well as the expectation $E(P)$ introduced in (13) are corresponding to the statistical characteristics of the proportion indices being manipulated generally in anthropometry.

The following holds

$$D(P) = \int [p - E(P)]^2 \, g(p) \, dp$$

After substitution for $g(p)$ from the relation

$$g(p) = \int g(p \mid x) f(x) \, dx$$

and after interchange of integrations we have

$$D(P) = \int \{ \int [p - E(P \mid x)]^2 g(p \mid x) \, dp +$$
$$+ \int [E(P \mid x) - E(P)]^2 g(p \mid x) \, dp \} f(x) \, dx$$

Range of integration of the variable x is divided exhaustively into a system of disjoint intervals x_i ($i = 1, 2, \ldots, q$) and the function $f(x)$ is replaced in each of them according to the approximation (10.) The following approximative expression is derived by it

$$D(P) = \frac{1}{2\Delta_X} \sum_{i=1}^{q} P(x_i) \left\{ \int_{x_i - \Delta_X}^{x_i + \Delta_X} D(P \mid x) \, dx + \right.$$
$$\left. + \int_{x_i - \Delta_X}^{x_i + \Delta_X} [E(P \mid x) - E(P)]^2 \, dx \right\}$$

and after substitution from (7) and (8) for $E(P \mid x)$ and $D(P \mid x)$ and from (13) for $E(P)$

$$D(P) = \frac{100^2}{2\Delta_X} \sum_{i=1}^{q} P(x_i) \left\{ \sigma_{W.X} \int_{x_i - \Delta_X}^{x_i + \Delta_X} \frac{1}{x^2} \, dx + \right.$$
$$+ (\mu_W - \beta_{W.X} \mu_X)^2$$
$$\left. \int_{x_i - \Delta_X}^{x_i + \Delta_X} \left[\frac{1}{x} - \frac{1}{2\Delta_X} \sum_{i=1}^{q} P(x_i) \ln \frac{x_i + \Delta_X}{x_i - \Delta_X} \right]^2 dx \right\}$$

Integration yields the following approximative formula for the variance of proportion

$$D(P) = 100^2 \left\{ \sigma_{W.X} \sum_{i=1}^{q} P(x_i) \frac{1}{(x_i + \Delta_X)(x_i - \Delta_X)} + \right.$$
$$+ (\mu_W - \beta_{W.X} \mu_X)^2 \qquad (15)$$

$$\left[\sum_{i=1}^{q} P(x_i) \frac{1}{(x_i + \Delta_X)(x_i - \Delta_X)} - \frac{1}{(2\Delta_X)^2} \cdot \left(\sum_{i=1}^{q} P(x_i) \ln \frac{x_i + \Delta_X}{x_i - \Delta_X}\right)^2\right]\right\}$$

Comparison of (12) and (13) gives the formula

$$E(P) = \sum_{i=1}^{q} P(x_i)\, E(P \mid x_i) \qquad (16)$$

i.e., this approximation being applied expected proportion in the whole population equals to the mean of expected proportions in the partial subpopulations x_i.

Similar comparison of the formulae (14) and (15) makes it possible to find that

$$D(P) = \sum_{i=1}^{q} P(x_i)\, D(P / x_i) + \frac{100^2}{(2\Delta_X)^2} (\mu_W - \beta_{W.X}\mu_X)^2$$

$$\left\{\sum_{i=1}^{q} P(x_i) \left[\ln \frac{x_i + \Delta_X}{x_i - \Delta_X}\right]^2 - \left[\sum_{i=1}^{q} P(x_i) \ln \frac{x_i + \Delta_X}{x_i - \Delta_X}\right]^2\right\}$$

$$= \sum P(x_i)\, D(P / x_i) + \frac{100^2}{(2\Delta_X)^2} (\mu_W - \beta_{W.X}\mu_X)^2\, D\left(\ln \frac{x_i + \Delta_X}{x_i - \Delta_X}\right) \qquad (17)$$

Hence, total variability of the proportion P in the populations equals — the proposed approximation being applied — to the mean variability of the proportion in the subpopulations x_i plus a certain constant depending on the variability of the auxiliar unction $\ln[(x_i + \Delta_X) / (x_i - \Delta_X)]$.

Taking into account that it holds

$$\lim_{\Delta_X \to 0} \frac{1}{2\Delta_X} \ln \frac{x_i + \Delta_X}{x_i - \Delta_X} = \frac{1}{x_i} \qquad (18)$$

and

$$\lim_{\Delta_X \to 0} \frac{1}{(x_i + \Delta_X)(x_i - \Delta_X)} = \frac{1}{x_i^2}$$

with $\Delta_X \to 0$ (i.e., when the width of the interval x_i decreases to zero), the expressions (12) and (14) transform for narrow intervals x_i into (7) and (8) where the value x_i is to be substituted instead of x.

PARAMETER ESTIMATION IN THE PROBABILITY DISTRIBUTION OF THE DIMENSIONS X, W) FROM ANTHROPOMETRICAL DATA

In the previous pages normal (Gaussian) conditional probability distribution of the dimension W in dependence on the value of the principal body dimension X as well as normal distribution of this dimension X (or Y) have been considered. Parameters of the distributions must be estimated from the

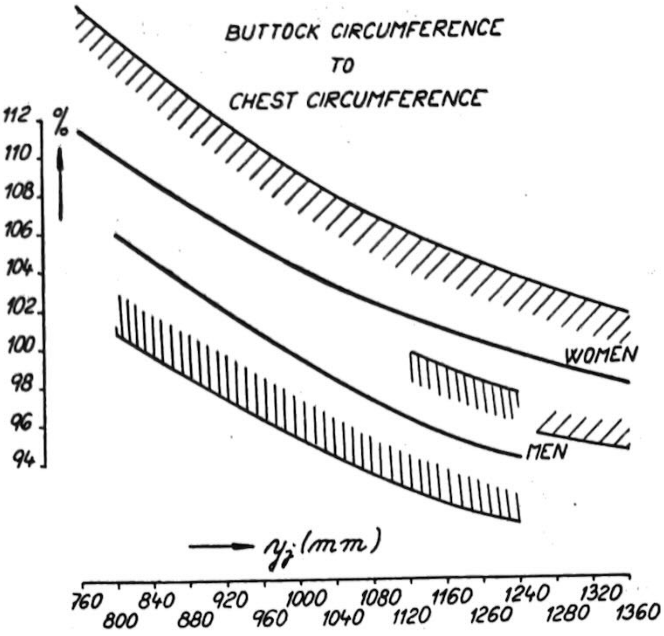

FIG. 2—18

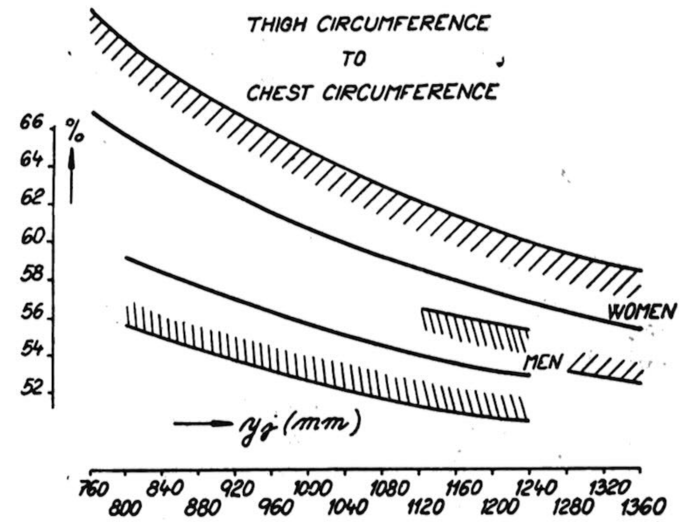

FIG. 2—19

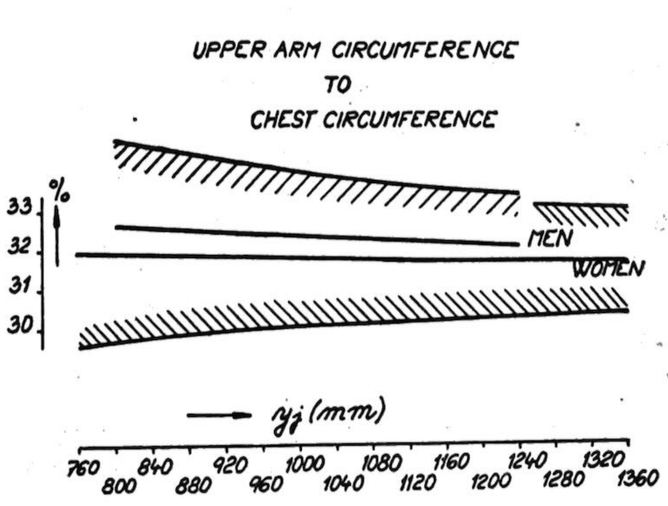

FIG. 2—20

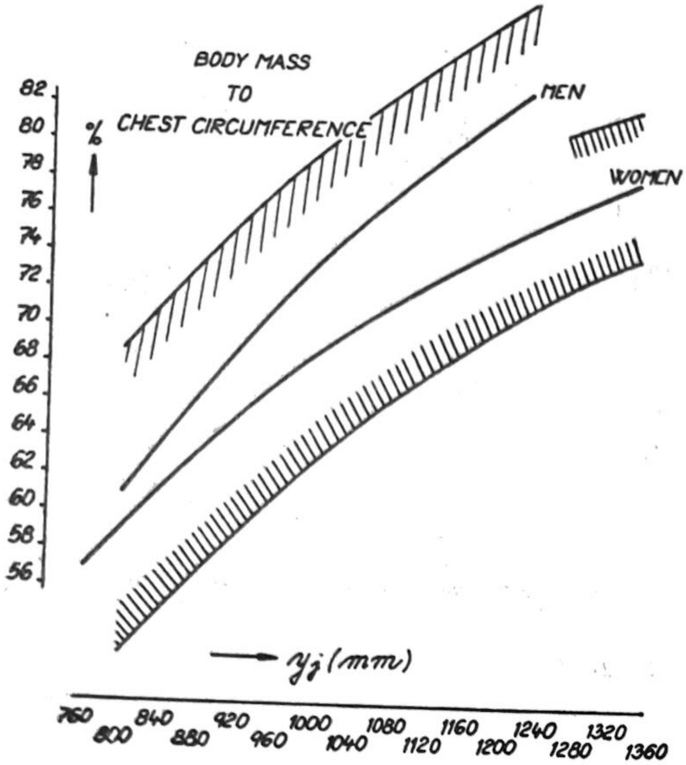

FIG. 2—21

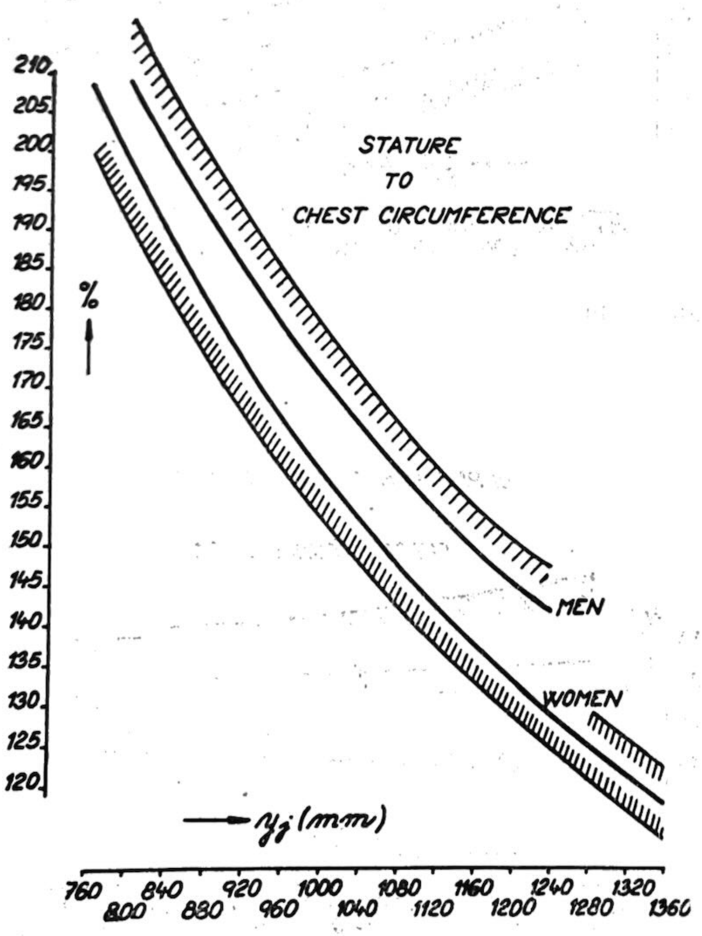

FIG. 2—22

anthropometrical data obtained in the measurement of body dimensions X (or Y) and W in a set of probands chosen randomly in the reference population. This material has been characterized in the introductory part of this communication.

Let us denote (x_α, w_α) concrete values of both body dimensions (X, W) in αth proband $(\alpha = 1, 2, \ldots, n)$, where n is the size of the set. Parameters of the theoretical probability distributions introduced above are estimated then by the following rules (the sign "$\hat{o}$" over the symbol of parameter means that an estimator is considered)

$$\hat{\mu}_X = \bar{x} = \frac{1}{n} \sum_{\alpha=1}^{n} x_\alpha \tag{19}$$

$$\hat{\mu}_W = \bar{w} = \frac{1}{n} \sum_{\alpha=1}^{n} w_\alpha \tag{20}$$

$$\hat{\sigma}_{XX} = s_X{}^2 = \frac{1}{n-1} \sum_{\alpha=1}^{n} (x_\alpha - \bar{x})^2 \tag{21}$$

$$\hat{\sigma}_{WW} = s_W{}^2 = \frac{1}{n-1} \sum_{\alpha=1}^{n} (w_\alpha - \bar{w})^2$$

$$\hat{\sigma}_{XW} = s_{XW} = \frac{1}{n-1} \sum_{\alpha=1}^{n} (x_\alpha - \bar{x})(w_\alpha - \bar{w}) \tag{23}$$

Estimators of the fundamental parameters thus obtained are summarized in *Table 2*.

In *Table 1* probabilities $P(x_i)$ or $P(y_j)$ of occurrence of probands in partial subpopulations x_i or y_j of the principal body dimensions X (stature) and Y (chest circumference) are given. In computation of $P(x_i)$ or $P(y_j)$ tables of standardized normal distribution have been used. Tolerance intervals $\Delta_X = 30$ mm and $\Delta_Y = 20$ mm have been chosen as adequate (see above).

In *Table 3* main numerical results are presented: expected proportions $E(P \mid x_i)$ or $E(P \mid y_j)$ and variances $D(P \mid x_i)$ or $D(P \mid y_j)$ in partial subpopulations x_i or y_j. In both last rows at the bottom of the table expected proportion $E(P)$ together with the variance $D(P)$ (and both its components) are given.

Inferior body dimensions W are those introduced in the text.

The dependence of expected proportions $E(P/x_i)$ or $E(P \mid y_j)$ on the categories x_i or y_j of the principal body dimension X or Y are demonstrated graphically, too. The range of $\pm \sqrt{D(P \mid x_i)}$ or $\sqrt{D(P \mid y_j)}$, i.e. of one S. D. over and below the mean is given for it. (see *Fig. 2*).

In *Fig. 1* histograms of theoretical frequencies of stature X and chest circumference Y are presented.

SUMMARY

Study of interrelations existing between body dimensions is one of the most important problems in physical anthropology. Proportion indices are belonging into this region of interest.

In the communication authors present an approximate method by means of which proportion indices can be computed without any knowledge of the index values for individual probands; only the values of statistical parameters of the frequency distribution in a set of probands being necessary.

Theoretical results are demonstrated in the case of 16 body dimensions interesting from the viewpoint of ergonomics, which have been measured in a set of 1,392 men and 1,373 women.

Table 1. Theoretical frequencies $P(x_i)$ and $P(y_j)$ of the categories x_i and y_j of stature X and chest circumference Y, respectively, in the population of men and women. The model of normal (Gaussian) probability distribution has been used in the computation, with the parameters

MEN	WOMEN
$\mu_X = 1{,}717$ mm,	$\mu_X = 1{,}596$ mm,
$\mu_Y = 1{,}015$ mm,	$\mu_Y = 969$ mm,
$\sigma_{XX} = 4{,}064$ mm²,	$\sigma_{XX} = 3{,}604$ mm²
$\sigma_{YY} = 5{,}303$ mm²,	$\sigma_{YY} = 8{,}234$ mm²

x_i(mm)	$P(x_i)$ MEN	$P(x_i)$ WOMEN
1,400		0.003
1,460		0.036
1,520	0.004	0.185
1,580	0.042	0.367
1,640	0.184	0.300
1,700	0.349	0.096
1,760	0.294	0.012
1,820	0.108	0.001
1,880	0.018	
1,940	0.001	
Σ	1.000	1.000

y_j(mm)	$P(y_j)$ MEN	$P(y_j)$ WOMEN
760		0.019
800		0.032
840	0.013	0.064
880	0.041	0.109
920	0.096	0.150
960	0.165	0.174
1,000	0.213	0.164
1,040	0.204	0.129
1,080	0.145	0.084
1,120	0.077	0.045
1,160	0.031	0.020
1,200	0.009	0.007
1,240	0.002	0.002
1,280		0.001
Σ	1.000	1.000

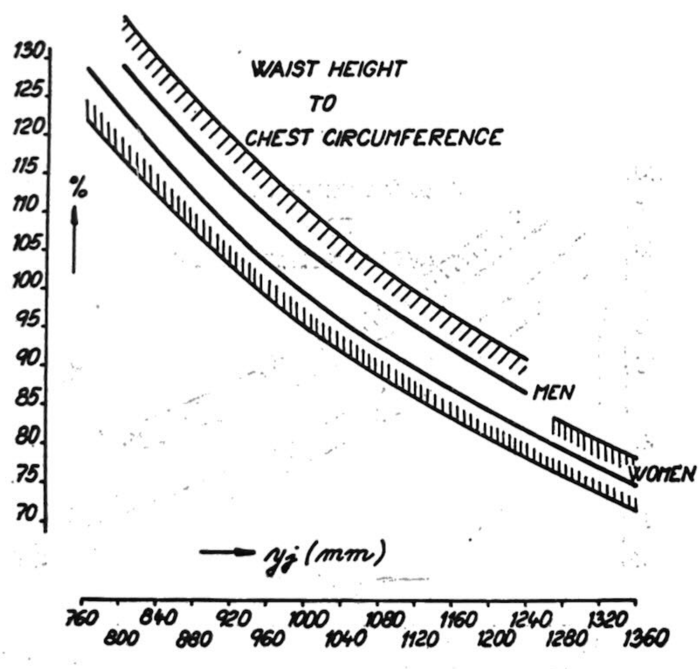

FIG. 2—23

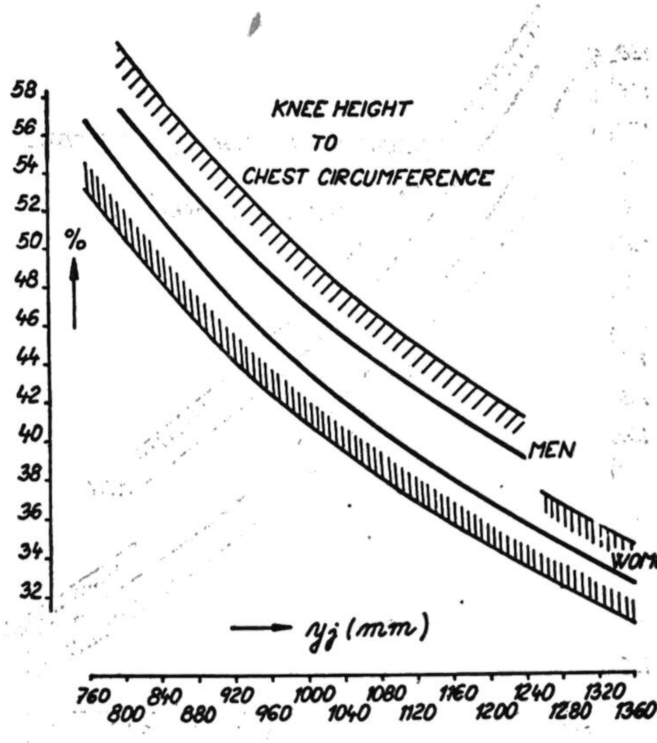

FIG. 2—24

Table 2. ***MEN. Fundamental statistical characteristics of body dimensions.***

W	$\bar{w}$	s_W^2	s_{XW}	r_{XW}	$b_{W.X}$	$s_{W.X}$
Neck C.	398	476	272	0.196	0.067	458
Waist C.	875	10,074	377	0.059	0.093	10,039
Buttock C.	1,004	4,468	1,239	0.291	0.305	4,090
Thigh C.	563	1,765	778	0.291	0.191	1,616
Upper Arm C.	329	791	235	0.131	0.058	777
Body Mass	75.0	116.27	316.1	0.460	0.778	91.68
Waist H.	1,051	2,687	2,994	0.907	0.737	480
Knee H.	473	750	1,333	0.764	0.328	313
H. 7th Neck V.	1,483	3,604	3,701	0.968	0.911	234
Glut. Fur. H.	785	1,809	2,275	0.840	0.560	535
Hip H.	864	2,126	2,364	0.805	0.582	751
Shoulder B.	398	494	539	0.381	0.133	423
Hip B.	345	464	624	0.455	0.154	368
Chest D.	255	469	247	0.179	0.061	454
Chest C.	1,015	5,303	1,006	0.217	0.248	5,054

W	$\bar{w}$	s_W^2	s_{YW}	r_{YW}	$b_{W.Y}$	$s_{W.Y}$
Neck C.	see above		1,121	0.706	0.211	239
Waist C.			6,042	0.827	1.139	3,189
Buttock C.			3,835	0.788	0.723	1,695
Thigh C.			2,178	0.712	0.411	870
Uper Arm C.			1,655	0.808	0.312	274
Body Mass			648.3	0.826	1.223	37.01
Waist H.			518	0.137	0.098	2,636
Knee H.			313	0.157	0.059	732
H. 7th Neck V.			1,171	0.268	0.221	3,345
Glut. Fur. H.			519	0.168	0.098	1,758
Hip H.			578	0.172	0.109	2,063
Shoulder B.			882	0.545	0.166	347
Hip B.			1,066	0.680	0.201	250
Chest D.			1,277	0.810	0.241	161
Stature	1,717	4,064	1,006	0.217	0.190	3,873

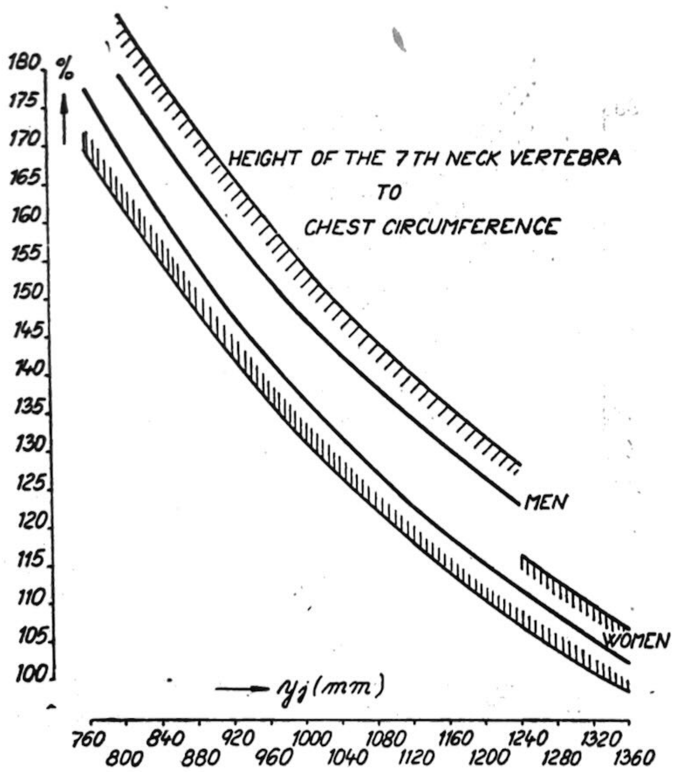

FIG. 2—25

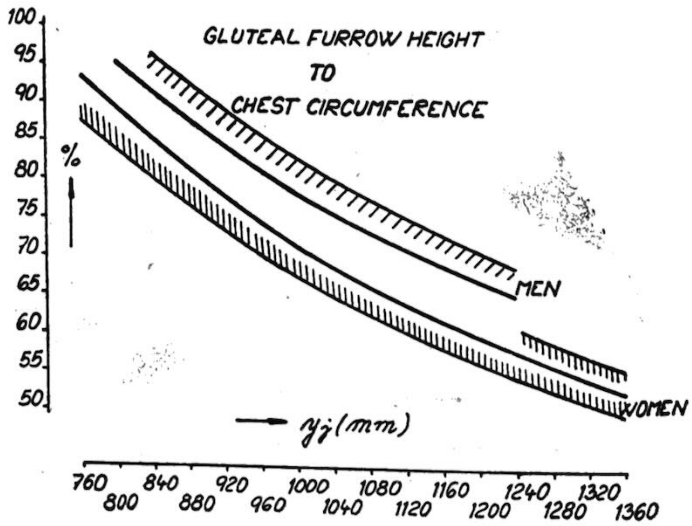

FIG. 2—26

TABLE 2. *WOMEN. Fundamental statistical characteristics (mean value, variance, covariance, correlation and regression coefficients and residual variance) of body dimensions*

W	$\bar{w}$	s_W^2	s_{XW}	r_{XW}	$b_{W.X}$	$s_{W.X}$
Neck C.	365	545	149	0.106	0.041	539
Waist C.	760	11,482	355	0.055	0.099	11,447
Buttock C.	1,016	7,800	604	0.114	0.168	7,699
Thigh C.	594	2,910	481	0.149	0.133	2,856
Upper Arm C.	309	1,157	46	0.023	0.013	1,156
Body Mass	65.1	119.43	166.9	0.255	0.463	111.70
Waist H.	992	2,252	2,509	0.881	0.696	505
Knee H.	437	773	1,186	0.711	0.329	383
H. 7th Neck V.	1,368	3,198	3,279	0.966	0.910	215
Glut. Fur. H.	713	1,693	2,126	0.861	0.590	439
Hip H.	796	2,155	2,118	0.760	0.588	910
Shoulder B.	356	292	421	0.411	0.117	243
Hip B.	353	775	368	0.220	0.102	737
Chest D.	264	870	65	0.037	0.018	869
Chest C.	969	8,234	206	0.038	0.057	8,222

W	$\bar{w}$	s_W^2	s_{YW}	r_{YW}	$b_{W.Y}$	$s_{W.Y}$
Neck C.			1,402	0.662	0.170	306
Waist C.			8,574	0.882	1.041	2,554
Buttock C.			6,661	0.832	0.809	7,747
Thigh C.			3,363	0.687	0.408	1,536
Upper Arm C.	see		2,592	0.840	0.315	1,498
Body Mass	above		856.6	0.864	1.040	30.32
Waist H.			539	0.125	0.065	2,217
Knee H.			171	0.068	0.021	538
H. 7th Neck V.			568	0.111	0.069	3,159
Glut. Fur. H.			57	0.015	0.007	1,693
Hip H.			495	0.118	0.060	1,831
Shoulder B.			690	0.445	0.084	234
Hip B.			1,718	0.680	0.209	417
Chest D.			2,399	0.897	0.291	171
Stature	1,596	3,604	206	0.038	0.025	3,599

Footnote: All body dimensions measured in mm, except of body mass where $\bar{w}$ is in kg, s_W^2 and $s_{W.X}$ and $s_{W.Y}$ are in kg², s_{XW} and s_{YW} are in cm.kg, $b_{W.X}$ and $b_{W.Y}$ are in kg cm^{-1}, r_{XW} and r_{YW} being dimensionless.

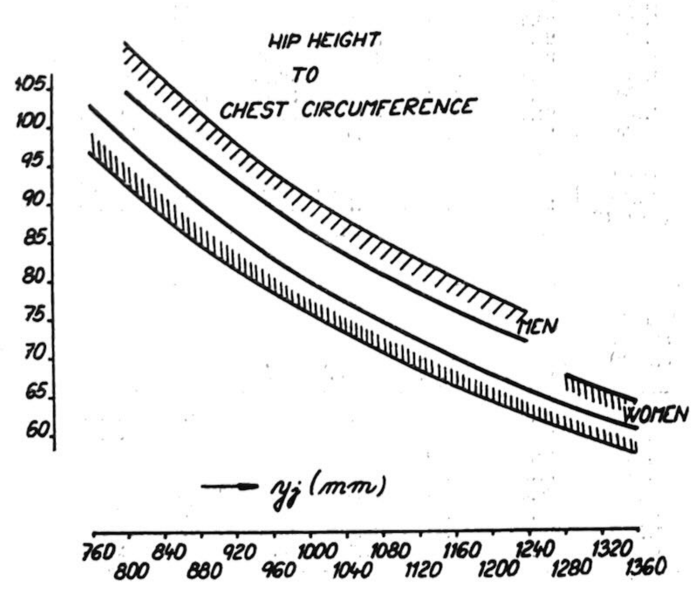

FIG. 2—27

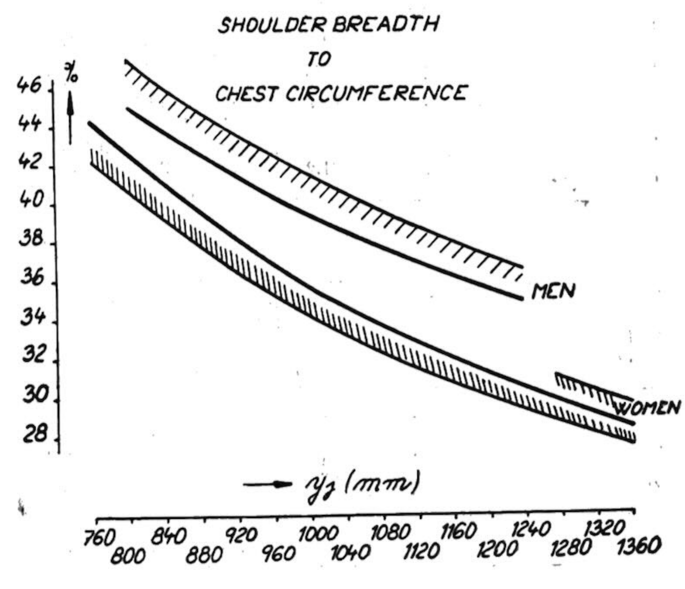

FIG. 2—28

Table 3. Expected proportions $E(P \mid x_i)$ or $E(P \mid y_j)$ and variances $D(P \mid x_i)$ or $D(P \mid y_j)$ of inferior body dimensions W to stature X or chest circumference Y, in particular subpopulations x_i or y_j. In the lower part of tables expectations $E(P)$ and variances $D(P)$ (as well as both its components $D_1(P)$ and $D_2(P)$) of proportion in the whole population are given. Proportions are dimensionless for all body dimensions W except of body mass, whose proportions are in kg . cm^{-1} .

MEN Stature x_i(mm)	Neck circumference		Waist circumference		Buttock circumference	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,520	25.3	2.03	56.4	43.76	62.1	17.84
1,580	24.6	1.87	54.6	40.47	60.9	16.50
1,640	24.0	1.74	52.9	37.55	59.8	15.31
1,700	23.3	1.61	51.4	34.93	58.8	14.24
1,760	22.8	1.50	49.9	32.58	57.8	13.28
1,820	22.2	1.40	48.6	30.45	56.9	12.41
1,880	21.8	1.32	47.4	28.54	56.0	11.63
1,940	21.3	1.23	46.2	26.75	55.3	10.90
$D_1(P) + D_2(P)$	—	1.59+0.41	—	31.42+2.59	—	14.02+1.17
$E(P)$ $D(P)$	23.2	1.99	51.0	34.01	58.5	15.19

MEN Stature x_i (mm)	Thigh circumference		Upper arm circumference		Chest circumference	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,520	34.6	7.03	20.9	3.40	63.6	22.08
1,580	34.0	6.50	20.3	3.14	62.1	20.42
1,640	33.4	6.03	19.8	2.91	60.7	18.94
1,700	32.9	5.61	19.3	2.71	59.5	17.62
1,760	32.5	5.24	18.8	2.53	58.3	16.43
1,820	32.0	4.89	18.4	2.36	57.2	15.36
1,880	31.6	4.59	18.0	2.21	56.1	14.39
1,940	31.2	4.30	17.6	2.07	55.2	13.48
$D_1(P) + D_2(P)$	—	5.53+0.28	—	2.67+0.27	—	17.34+1.76
$E(P)$ $D(P)$	32.8	5.81	19.2	2.94	59.2	19.10

MEN Stature x_i (mm)	Body mass		Waist height		Knee height	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$E(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,520	39.3	39.89	59.6	2.10	26.9	1.36
1,580	40.7	36.90	60.1	1.94	27.1	1.26
1,640	42.1	34.24	60.6	1.80	27.3	1.17
1,700	43.3	31.86	61.1	1.68	27.5	1.09
1,760	44.5	29.71	61.5	1.56	27.7	1.01
1,820	45.6	27.78	61.9	1.46	27.8	0.95
1,880	46.6	26.03	62.3	1.37	28.0	0.89
1,940	47.6	24.41	62.6	1.28	28.2	0.83
$D_1(P) + D_2(P)$	—	31.36+1.74	—	1.65+0.23	—	1.07+0.04
$E(P)$ $D(P)$	43.6	33.10	61.2	1.88	27.5	1.11

MEN Stature x_i(mm)	Height of 7th neck vertebra		Gluteal furrow height		Hip height	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,520	85.8	1.02	44.4	2.34	49.3	3.26
1,580	86.0	0.94	44.8	2.16	49.6	3.02
1,640	86.2	0.87	45.2	2.00	50.0	2.80
1,700	86.3	0.81	45.6	1.86	50.2	2.61
1,760	86.5	0.76	46.0	1.74	50.5	2.43
1,820	86.6	0.71	46.3	1.63	50.8	2.27
1,880	86.8	0.66	46.6	1.52	51.0	2.13
1,940	86.9	0.62	46.9	1.43	51.2	2.00
$D_1(P) + D_2(P)$	—	0.80+0.03	—	1.83+0.16	—	2.57+0.09
$E(P)$ $D(P)$	86.4	0.83	45.7	1.99	50.3	2.66

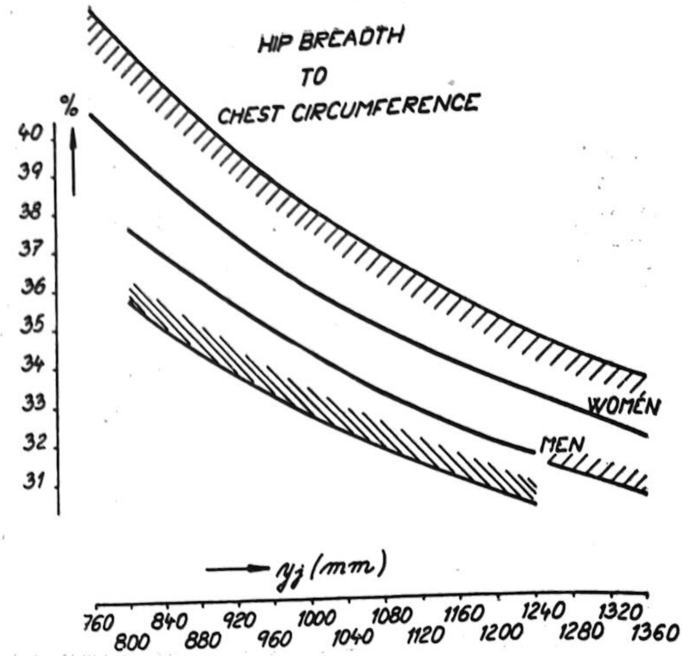

FIG. 2—29

TABLE 3. continued

MEN Stature x_i(mm)	Shoulder breadth		Hip breadth		Chest depth	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,520	24.5	1.85	20.7	1.60	16.0	1.98
1,580	24.0	1.71	20.5	1.48	15.6	1.83
1,640	23.6	1.58	20.3	1.37	15.3	1.70
1,700	23.3	1.47	20.1	1.28	14.9	1.58
1,760	22.9	1.37	20.0	1.19	14.6	1.47
1,820	22.6	1.28	19.8	1.11	14.4	1.38
1,880	22.3	1.20	19.7	1.04	14.1	1.29
1,940	22.0	1.13	19.5	0.98	13.8	1.21
$D_1(P) + D_2(P)$	—	1.45+0.15	—	1.26+0.03	—	1.55+0.11
$E(P)$ $D(P)$	23.2	1.60	20.1	1.29	14.8	1.66

WOMEN Stature x_i(mm)	Neck circumference		Waist circumference		Buttock circumference	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,400	25.5	2.82	52.9	58.71	70.2	39.73
1,460	24.6	2.59	51.1	53.96	68.0	36.50
1,520	23.8	2.38	49.5	49.77	66.0	33.65
1,580	23.1	2.20	48.0	46.05	64.1	31.12
1,640	22.4	2.04	46.6	42.73	62.4	28.87
1,700	21.7	1.90	45.3	39.75	60.8	26.85
1,760	21.1	1.77	44.1	37.08	59.3	25.03
1,820	20.6	1.65	43.0	34.67	57.9	23.40
$D_1(P) + D_2(P)$	—	2.17+0.55	—	45.34+2.23	—	30.64+3.44
$E(P)$ $D(P)$	22.9	2.72	47.7	47.57	63.7	34.08

WOMEN Stature x_i(mm)	Thigh circumference		Upper arm circumference		Chest circumference	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,400	40.6	14.64	21.9	5.97	68.4	42.57
1,460	39.4	13.45	21.0	5.48	65.8	39.10
1,520	38.4	12.40	20.3	5.05	63.5	36.03
1,580	37.5	11.47	19.5	4.67	61.3	33.32
1,640	36.6	10.65	18.9	4.34	59.2	30.90
1,700	35.8	9.90	18.3	4.03	57.4	28.74
1,760	35.0	9.23	17.7	3.76	55.6	26.79
1,820	34.3	8.63	17.1	3.51	53.9	25.04
$D_1(P) + D_2(P)$	—	11.30+0.89	—	4.60+0.51	—	32.81+4.73
$E(P)$ $D(P)$	37.3	12.19	19.4	5.11	60.8	37.54

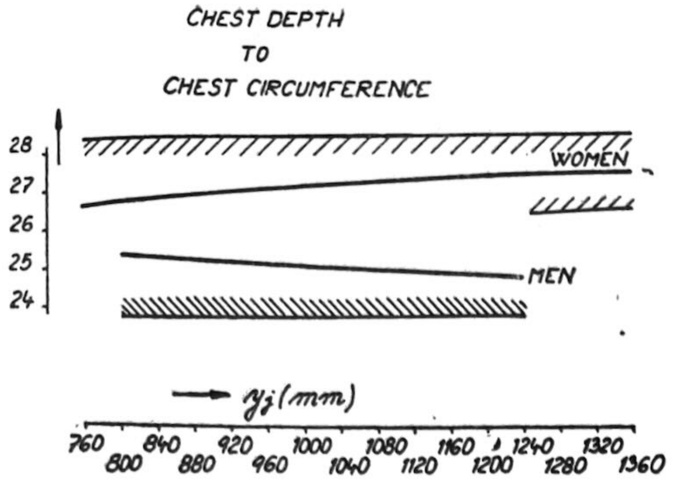

FIG. 2—30

TABLE 3. continued

WOMEN x_i(mm)	Body mass		Waist height		Knee height	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,400	40.0	57.02	61.1	2.59	26.6	1.96
1,460	40.3	52.43	61.5	2.38	26.9	1.80
1,520	40.5	48.37	61.8	2.20	27.1	1.66
1,580	40.7	44.76	62.1	2.03	27.3	1.54
1,640	40.9	41.55	62.4	1.89	27.5	1.43
1,700	41.1	38.67	62.6	1.75	27.7	1.33
1,760	41.3	36.07	62.9	1.64	27.9	1.24
1,820	41.5	33.73	63.1	1.53	28.1	1.16
$D_1(P) + D_2(P)$	–	44.08+0.05	–	2.00+0.09	–	1.52+0.05
$E(P)$ $D(P)$	40.8	44.13	62.2	2.09	27.4	1.57

WOMEN Stature x_i(mm)	Height of 7th neck vertebra		Gluteal furrow height		Hip height	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,400	85.0	1.10	42.7	2.28	48.6	4.66
1,460	85.2	1.01	43.3	2.09	49.0	4.29
1,520	85.5	0.93	44.0	1.93	49.4	3.95
1,580	85.7	0.86	44.5	1.78	49.8	3.66
1,640	85.9	0.80	45.1	1.65	50.1	3.39
1,700	86.0	0.75	45.5	1.54	50.4	3.16
1,760	86.2	0.70	46.0	1.43	50.7	2.95
1,820	86.4	0.65	46.4	1.34	51.0	2.75
$D_1(P) + D_2(P)$	–	0.85+0.04	–	1.75+0.32	–	3.60+0.12
$E(P)$ $D(P)$	85.7	0.89	44.7	2.07	49.8	3.72

WOMEN Stature x_i(mm)	Shoulder breadth		Hip breadth		Chest depth	
	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$	$E(P \mid x_i)$	$D(P \mid x_i)$
1,400	23.8	1.26	23.8	3.79	18.6	4.48
1,460	23.3	1.16	23.2	3.48	17.9	4.11
1,520	22.8	1.07	22.7	3.21	17.3	3.79
1,580	22.4	0.99	22.2	2.97	16.7	3.51
1,640	22.0	0.92	21.8	2.76	16.1	3.25
1,700	21.7	0.85	21.4	2.57	15.6	3.03
1,760	21.3	0.79	21.0	2.39	15.2	2.82
1,820	21.0	0.74	20.7	2.24	14.7	2.64
$D_1(P) + D_2(P)$	–	0.98+0.18	–	2.93+0.22	–	3.45+0.34
$E(P)$ $D(P)$	22.3	1.16	22.1	3.15	16.6	3.79

TABLE 3. continued

MEN Chest circumference y_j(mm)	Neck circumference		Waist circumference		Buttock circumference	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
800	44.1	3.85	78.7	50.13	106.1	26.73
840	43.0	3.48	80.4	45.45	104.5	24.22
880	42.0	3.16	81.9	41.39	103.0	22.06
920	41.1	2.89	83.3	37.86	101.7	20.17
960	40.2	2.65	84.6	34.75	100.4	18.51
1,000	39.5	2.44	85.8	32.02	99.3	17.05
1,040	38.8	2.25	86.9	29.59	98.3	15.76
1,080	38.1	2.08	87.9	27.44	97.3	14.60
1,120	37.5	1.93	88.8	25.51	96.4	13.57
1,160	37.0	1.80	89.7	23.77	95.6	12.65
1,200	36.4	1.68	90.5	22.20	94.8	11.81
1,240	35.9	1.57	91.2	20.79	94.1	11.06
$D_1(P) + D_2(P)$	–	2.41+1.92	–	31.64+4.52	–	16.85+4.17
$E(P)$ $D(P)$	39.3	4.33	86.0	36.16	99.1	21.02

MEN Chest circumference y_j(mm)	Thigh circumference		Upper arm circumference		Body mass	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
800	59.3	13.68	32.7	4.29	60.9	58.65
840	58.5	12.40	32.7	3.89	63.8	53.13
880	57.7	11.29	32.6	3.55	66.5	48.36
920	57.0	10.33	32.5	3.24	68.9	44.20
960	56.3	9.48	32.5	2.98	71.1	40.56
1,000	55.7	8.74	32.4	2.75	73.2	37.55
1,040	55.1	8.08	32.4	2.54	75.0	34.51
1,080	54.6	7.49	32.3	2.35	76.8	31.98
1,120	54.1	6.96	32.3	2.19	78.4	29.72
1,160	53.7	6.49	32.3	2.04	79.9	27.69
1,200	53.2	6.06	32.2	1.91	81.3	25.85
$D_1(P) + D_2(P)$	–	8.63+1.22	–	2.71+0.01	–	36.90+13.79
$E(P)$ $D(P)$	55.6	9.85	32.4	2.72	73.6	50.69

MEN Chest circumference y_j(mm)	Stature		Waist height		Knee height	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
800	209.6	68.11	128.8	44.16	57.5	11.99
840	200.5	61.13	123.1	39.80	55.1	10.83
880	192.2	55.20	118.0	36.07	52.9	9.83
920	184.7	50.13	113.2	32.86	50.8	8.97
960	177.8	45.69	108.9	30.04	48.9	8.21
1,000	171.4	41.84	105.0	27.58	47.2	7.55
1,040	165.6	38.47	101.3	25.42	45.6	6.96
1,080	160.1	35.47	97.9	23.49	44.2	6.44
1,120	155.1	32.84	94.8	21.79	42.8	5.98
1,160	150.4	30.51	91.8	20.27	41.5	5.56
1,200	146.0	28.23	89.1	18.83	40.3	5.18
1,240	141.9	26.43	86.5	17.63	39.2	4.85
$D_1(P) + D_2(P)$		−41.37+132.39		−27.26+51.85	–	7.46+9.76
$E(P)$ $D(P)$	170.1	173.76	104.1	79.11	46.9	17.22

MEN Chest circumference y_j(mm)	Height of 7th neck vertebra		Gluteal furrow height		Hip height	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
800	179.5	57.45	95.5	29.02	105.1	34.10
840	172.0	51.67	91.4	26.19	100.6	30.77
880	165.2	46.74	87.7	23.76	96.5	27.91
920	158.9	42.51	84.3	21.66	92.8	25.45
960	153.2	38.80	81.2	19.82	89.4	23.29
1,000	148.0	35.58	78.4	18.21	86.2	21.39
1,040	143.1	32.75	75.7	16.80	83.3	19.73
1,080	138.7	30.23	73.3	15.54	80.7	18.24
1,120	134.5	28.01	71.0	14.42	78.2	16.93
1,160	130.6	26.04	68.9	13.42	75.9	15.76
1,200	127.0	24.14	66.9	12.48	73.7	14.65
1,240	123.6	22.60	65.1	11.69	71.7	13.72
$D_1(P) + D_2(P)$		−35.17+90.35		−18.00+26.92		−21.14+32.44
$E(P)$ $D(P)$	146.9	125.52	77.7	44.92	85.6	53.58

TABLE 3. continued

MEN Chest circumference y_j(mm)	Shoulder breadth		Hip breadth		Chest depth	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
800	45.3	5.60	37.7	3.97	25.4	2.53
840	43.9	5.07	36.9	3.59	25.3	2.29
880	42.7	4.60	36.1	3.27	25.3	2.09
920	41.5	4.20	35.4	2.99	25.2	1.91
960	40.5	3.85	34.8	2.74	25.2	1.75
1,000	39.6	3.54	34.2	2.52	25.1	1.62
1,040	38.7	3.27	33.7	2.33	25.1	1.49
1,080	37.9	3.03	33.2	2.16	25.1	1.39
1,120	37.1	2.81	32.7	2.01	25.0	1.29
1,160	36.4	2.62	32.3	1.87	25.0	1.20
1,200	35.7	2.44	31.9	1.75	25.0	1.12
1,240	35.1	2.29	31.5	1.64	24.9	1.05
$D_1(P) + D_2(P)$	–	3.50+3.03	–	2.49+1.14	–	1.60+0.01
$E(P)$ $D(P)$	39.4	6.53	34.1	3.63	25.1	1.61

WOMEN Chest circumference y_j(mm)	Stature		Waist height		Knee height	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
760	209.4	72.23	128.8	41.85	56.9	14.03
800	199.0	64.30	122.6	37.46	54.2	12.59
840	189.7	57.63	117.1	33.74	51.7	11.38
880	181.1	51.98	112.1	30.55	49.5	10.33
920	173.4	47.16	107.5	27.81	47.4	9.42
960	166.3	42.95	103.3	25.42	45.5	8.63
1,000	159.7	39.29	99.4	23.32	43.8	7.93
1,040	153.7	36.10	95.8	21.49	42.2	7.31
1,080	148.1	33.26	95.2	19.85	40.7	6.77
1,120	142.9	30.77	89.5	18.40	39.3	6.28
1,160	138.0	28.58	86.6	17.12	38.0	5.85
1,200	133.5	26.41	83.9	15.89	36.8	5.44
1,240	129.3	24.72	81.4	14.88	35.7	5.10
1,280	125.3	23.18	79.1	13.96	34.6	4.78
1,320	121.6	21.67	76.9	13.08	33.7	4.49
1,360	118.1	20.33	74.8	12.29	32.7	4.22
$D_1(P) + D_2(P)$	–	43.44+244.65	–	25.67+85.35	–	8.71+17.20
$E(P)$ $D(P)$	166.1	288.09	103.2	111.02	45.5	25.91

WOMEN Chest circumference y_j(mm)	Neck circumference		Waist circumference		Buttock circumference	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
760	43.3	5.47	71.4	44.50	114.4	41.99
800	42.0	4.92	73.0	40.13	109.9	37.88
840	40.8	4.45	74.5	36.38	108.5	34.34
880	39.8	4.05	75.8	33.13	107.3	31.28
920	38.8	3.70	77.1	30.30	106.1	28.61
960	37.9	3.39	78.2	27.82	105.1	26.26
1,000	37.0	3.12	79.2	25.63	104.1	24.20
1,040	36.3	2.88	80.2	23.69	103.2	22.37
1,080	35.5	2.67	81.1	21.96	102.4	20.73
1,120	34.9	2.48	81.9	20.42	101.6	19.28
1,160	34.3	2.31	82.7	19.03	100.9	17.97
1,200	33.7	2.15	83.4	17.78	100.2	16.78
1,240	33.2	2.01	84.0	16.65	99.6	15.72
1,280	32.7	1.89	84.7	15.62	99.0	14.75
1,320	32.2	1.77	85.3	14.69	98.5	13.87
1,360	31.7	1.67	85.8	13.83	98.0	13.06
$D_1(P) + D_2(P)$	–	3.42+3.96	–	28.03+6.13	–	26.46+5.32
$E(P)$ (DP)	36.9	7.38	79.5	34.16	100.8	31.78

WOMEN Chest circumference y_j(mm)	Height of 7th neck vertebra		Gluteal furrow height		Hip height	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
760	178.1	61.50	93.6	31.32	103.1	39.00
800	169.6	54.89	89.0	28.08	98.2	35.00
840	161.8	49.31	84.8	25.33	93.9	31.59
880	154.8	44.57	81.0	22.98	89.9	28.67
920	148.4	40.50	77.5	20.94	86.2	26.14
960	142.5	36.95	74.3	19.16	82.9	23.92
1,000	137.0	33.85	71.3	17.60	79.8	21.98
1,040	132.0	31.15	68.6	16.22	77.0	20.28
1,080	127.4	28.73	66.1	15.00	74.3	18.75
1,120	123.1	26.61	63.8	13.92	71.9	17.40
1,160	119.1	24.73	61.6	12.95	69.6	16.20
1,200	115.3	22.91	59.6	12.04	67.5	15.07
1,240	111.8	21.45	57.7	11.28	65.5	14.11
1,280	108.6	20.11	55.9	10.58	63.7	13.24
1,320	105.5	18.83	54.2	9.92	61.9	12.42
1,360	102.6	17.67	52.6	9.33	60.3	11.68
$D_1(P) + D_2(P)$	–	37.34+167.57	–	19.34+49.29	–	24.14+53.86
$E(P)$ $D(P)$	142.4	204.91	74.2	68.63	82.8	78.00

WOMEN Chest circumference y_j(mm)	Thigh circumference		Upper arm circumference		Body mass	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
760	66.9	26.78	32.0	5.91	57.0	53.03
800	65.6	24.15	32.0	5.32	59.4	47.81
840	64.4	21.89	32.0	4.84	61.5	43.33
880	63.4	19.94	31.9	4.41	63.4	39.45
920	62.4	18.23	31.9	4.03	65.2	36.07
960	61.5	16.74	31.9	3.70	66.8	33.11
1,000	60.7	15.42	31.9	3.41	68.3	30.50
1,040	59.9	14.26	31.9	3.15	69.7	28.18
1,080	59.2	13.22	31.8	2.93	71.0	26.12
1,120	58.5	12.29	31.8	2.72	72.1	24.28
1,160	57.9	11.45	31.8	2.54	73.2	22.63
1,200	57.4	10.70	31.8	2.37	74.3	21.13
1,240	56.8	10.02	31.8	2.22	75.2	19.79
1,280	56.3	9.40	31.8	2.08	76.1	18.57
1,320	55.9	8.84	31.8	1.96	77.0	17.46
1,360	55.4	8.32	31.8	1.84	77.8	16.44
$D_1(P) + D_2(P)$	–	16.87+3.92	–	3.73+0.00	–	33.37+12.60
$E(P)$ $D(P)$	61.5	20.79	31.9	3.73	66.8	45.97

WOMEN Chest circumference y_j(mm)	Shoulder breadth		Hip breahth		Chest depth	
	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$	$E(P \mid y_j)$	$D(P \mid y_j)$
760	44.5	4.36	40.7	7.31	26.7	2.96
800	42.7	3.91	39.7	6.59	26.8	2.68
840	41.1	3.52	38.8	5.97	27.0	2.43
880	39.6	3.19	38.0	5.43	27.1	2.21
920	38.3	2.91	37.3	4.97	27.1	2.02
960	37.0	2.66	36.6	4.56	27.2	1.86
1,000	35.9	2.44	35.9	4.20	27.3	1.71
1,040	34.8	2.25	35.4	3.88	27.4	1.58
1,080	33.8	2.08	34.8	3.59	27.4	1.47
1,120	32.9	1.93	34.3	3.34	27.5	1.36
1,160	32.1	1.80	33.9	3.11	27.6	1.27
1,200	31.3	1.67	33.4	2.91	27.6	1.19
1,240	30.5	1.56	33.0	2.72	27.7	1.11
1,280	29.9	1.47	32.6	2.55	27.7	1.04
1,320	29.2	1.38	32.3	2.40	27.7	0.98
1,360	28.6	1.29	32.0	2.26	27.8	0.93
$D_1(P) + D_2(P)$	–	2.71+7.32	–	4.60+2.23	–	1.87+0.03
$E(P)$ $D(P)$	37.0	10.03	34.8	6.83	27.2	1.90

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