ANALYTICAL DESCRIPTION OF CRANIAL PROFILES BY MEANS OF KTH ORDER POLYNOMIAL EQUATIONS

PROCEDURE AND APPLICATION ON PLESIANTHROPUS TRANSVAALENSIS (ST55)

ABSTRACT — A method to describe skull profiles by means of analytical morphometrical evaluators aiming at systematic comparisons and divergence ranking is presented. The technique used is fully automated and employs a television camera interfaced with a computer having specific hardware devices and an original S. A. M. (Shape Analytical Morphometry) software package. The main routines provide both coefficients of the upper degree equation, their standard deviation and standard error, as well as variance and covariance matrices whereas accessory routines supply both standardization of positioning, normalization of the apparent size of the image to be analyzed and regulations for densitometric readings. The five upper degree polynomials for the profiles of the total i.e. frontal-facial, facial, frontal and corresponding caudal trait of the left side of the Australopithecus africanus (ST55, Plesianthropus transvaalensis) together with its sagittal curve are discussed in detail.

KEY WORDS: Skull profiles — Upper degree polynomial equations — Analytical morphology.

INTRODUCTION

One of the most important goals in morphological studies is to find descriptive methods which can reduce the observer’s subjectivity and facilitate comparisons through parametrization. As stated by D’Arcy W. Thompson: “Problems of form are in the first instance mathematical problems, ... problems of growth are essentially physical problems...” With these words, in 1917, he emphasized the nature of quantitative morphology himself if had helped to found. Fifty-seven years later, P. Dommernek confirmed that: “…mathematical tools are methods; they serve for the analysis, the discrimination or the generalisation, but the explanation is typological, causal or historical rather than mathematical. A true mathematical idea implies a formulation of the relations of shape and form to other phenomena in terms of exactly defined mathematical parameters.”

In the words of D’Arcy Thompson and Dommernek we find the principles of rigor and essentiality such which should be applied in describing the human form. Both authors draw attention to the connection between description and structural and functional interpretations which seem to be the two keystones in the thought and works of S. Sergi. Indeed these are the basic premises needed to reach the ultimate formalizations of the description.

Some methodological aspects of the problem have been studied by Osmuz (1973, 1984), Lestrel (1974) and more recently by Ignaill et al. (1986).

As underlined by Jacobshagen (1928), the quantification of irregular organic structures is limited if approached through traditional method, i.e. utilizing measurements taken between discreet anatomical points, since from a morphological point of view a relevant quantity of information can be lost. Classical morphometry is currently utilizing measurements and their derive fractions with practical purposes, but none of these parameters represent the description of a shape which should be obtained by mathematical functions independently from dimension (analytical morphometry).

In our work we have examined problems of
6. Motorized telerecontrolled stands for skulls and photographs with all the spatial movements.

7. Skulls of the same standard photographs. For the present study a lateral left view photograph (fig. 1) of the Pleistothorax transversalis (Broom et al. 1950) and as Du Brul (1974) was utilized.

Finally, the number of lines for the scanning were determined for the treatment of a particular part of the image. For dimensional normalization different standard numbers of number lines were used for the scanning in different situations. A standard of 150 lines was used for 1 total frontal-facial profile (vertex-prosition), 2 frontal profiles, and 3 posterior profile. For the reconstruction of the functional profile of the base from the glabella to the point in which the posterior profile crosses the horizontal plane where the glabella lies was divided in 230 lines, which was a standard of 870 lines had been adopted for frontal-facial profile, the standard was now optimized to 150 lines. The coordinates of these points were expressed in arbitrary units (screen units) represented by the smallest treatable part of the digitized image (pixel). They represent the resolution absolute level (the relative one varies with the factor of the image enlargement). The parameters used were: for the abscissa interval: $X_{min}$, $X_{max}$ 254 from left to right and for the ordinate interval: $Y_{min}$, $Y_{max}$ from top to bottom.

MATERIALS

The software utilized was the S. A. M. (Shape Analytical Morphometry) package specifically and originally implemented in our laboratory. This package includes more than 350 routines for different items of analytical morphometry, some analysis, image processing oriented to the study of biological structures. The software routine used can be grouped according to their specific function as follows:

1. Routine to control the system of coordinates and the image luminosity.

2. The computer first generated a system of luminous points with known coordinates on a monitor where the analog TV image and the image of the variable set were visualized. To ensure repeatability of the system of coordinates the net was calibrated so that a luminous point was situated at the continuity section between a given vertical with a given horizontal line (fig. 7). After processing a complete net was built by the software on the digitized image present on the computer monitor to verify the congruity between the analogic and the digitalized images.
Different profiles were mainly referred to different scale factors to optimize relative enlargement of image and resolution of scanning. Nevertheless software allowed to pass a printout of the profile of type 1, if same scale is required. For a real length of the worked profile of about 177 mm (for fronto-facial profile) and 109 mm (for palatal profile), the average resolution was about 93 mm. This value differs in different traits since the profile orientation to the scanning can vary. In terms of incremental trend of slope (point after point delta %) we observed a range from 8 to 9. During scanning a densitometric reading classifying the image into 64 grey levels was made. The aforementioned are some of the fundamental methods of "scene analysis" (pattern recognition, image processing) and graphical elaboration. These techniques can be applied in a number of fields where automatic recognition can be useful (Hall, 1979; Rosenfeld, 1982).

In the procedure adopted the densitometric reading was discriminated only between the classes of grey referred to either the background or to the image. These could be distinguished by a threshold value, and thus the value of the abscissa where the threshold was localized was registered. The luminosity, the interface contrast, and the illumination of the object under study were all controlled by intervening when necessary.

In utilizing the techniques discussed above the procedure was as follows:

(i) Selection of the skulls to be analyzed. Modern skulls are to be chosen to evaluate the differences due to the effect of the environment as well as differences due to age and sex. Fossil skulls are to be worked for comparisons of phylogenic relevance. Photographs should be taken of both medical norms or modern skulls or casts of fossil ones if can also be used.

(ii) Selection of the profile of the crania to be studied. Obviously this is possible only when the material consists of skulls or casts.

(iii) Standardization of position and normalization of dimensions of the specimen. The procedure of positioning depends on whether the specimens are real skulls, casts or photographs. For real skulls or casts there should be a standard procedure for positioning the postures changes are possible only in the plane parallel to the camera. For skulls and casts, the camera should be at least six meters from the cranial deformation. The plane of reference has to be defined in all cases.

In the present work we used the Frankfurt plane and the horizontal line coinciding with the use of a tuned analog net by making one of its characteristic lines coincide with the Frankfurt plane. Two values were obtained on the analog monitor and located on the same horizontal linear were localized at the extremities of the Frankfurt plane. The abscissa of profile was located at scanning line number one and the lower point of each profile was located at the ending point of the corresponding horizontal line. It must be noted that these two values of ordinates were common to all profiles of the same traits of different images; the abscissa values of the rightmost point of the caudal (at right) profile was also common to all (equal to 233 in our coordinate system). Consequently, a series of ordinates were obtained for different and different images were dimensionally normalized.

Actually the caudal profile alone was truly normalized because it had been positioned so as to make the average length of all different skulls caused non comparable positioning of the rostral (at left) profiles. This was solved by means of the software that allowed renormalization of the rostral profiles by simply transferring the first deepest point of the naso-glabella segment to a fixed value of the abscissa (equal to 100). So all dimensionally normalized differences were eliminated and only differences of shape were considered in subsequent analytical methods.

The final stage of the whole called exploration for the regulation of the illumination intensity so as to obtain good contrast between the image and the background. At this point it was possible to proceed with the image analysis routines.

2. ROUTINES FOR THE ALGORITHM APPLICATION TO DEFINE THE DESCRIPTIVE EQUATION.

These algorithms can be of various type: skeletonizing procedures useful mostly from a graphic point of view; polynomial and analysis of the frequency spectra. In regards to the polynomial among the numerous possibilities, namely natural cubic splines, normalized polynomials, and polynomials of an upper degree, we feel that the latter are the more useful. We will use, in general, polynomials or splines. They furnish a description by approximation, whereas the polynomial splines provide the exact matching for the series of points, interpolating exactly. The expression is kept very low in respect to the maximum possible degree which should correspond to the number of points minus one. The use of the algorithm of least squares of high degree polynomials.

This, together with the graphic control adopted method, has given good quality results though those were obtained through easy to handle equations. Moreover the smoothing effect introduced by the upper degree polynomial is particularly useful to smooth the curves in the skulls of which there are few samples or only one specimen available, as in the case of fossil remains. In fact the algorithm of only four equations limits the number of the methodological error procedures, it also reduces the influence of the morphological characteristic of least importance (individual ones) present in each cranial bone because of their gaussian distribution with respect to the function. On the contrary a periodic error distribution calls for a great deal of effort for the normalization of the curve. In any case we obtain an essential but characteristic curve for each worked profile.

The algorithm of the entire equation of the descriptive representation employed by a polynomial of K<sup>th</sup> order of the type,

\[ Y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \ldots + b_k x^k \]

is based on the least squares regression-interpolation method. If \((Y_i, X_i)\) is the \(i^{th}\) value observed, with \(Y_i\) (dependent variable) as the abscissa value of the point found by the scanning on the profile in correspondence to the fixed ordinate \(X_i\) (independent variable, for \(i = 1, 2, 3, \ldots n\) where \(n\) is maximum ordinate value), in each equation in the set (the standard of scanning), then the minimum of the following function with the variables \(b_0, b_1, \ldots b_k\) can be determined:

\[ \sum (y_i - b_0 - b_1 x_i - b_2 x_i^2 - \ldots - b_k x_i^k)^2 \]

so as to render null all the partial derivatives with respect to the polynomial coefficients. A partial description of the equation system is thus made up of \(K + 1\) unknown values where \(K + 1\) is the number of coefficients to determine in order to establish the unique solution (Cahn, 1955).

A fundamental problem was the selection of the independent variables. The scaling values of the independent variables were represented by the abscissa values of the profile single points found by the computer program. These were based on crossing a threshold (cut-off) value, defined by the operator according to image luminosity during densitometric scanning.

According to the value of the fit goodness for the function, (high values of the coefficient of determination and low values of the standard error and of the estimated variance) for an order of the polynomial which is acceptably low in respect to the number of points, the best equi-penalty integer order for the series between 1 and 5. From the top to the bottom of the profiles these series can be considered as ordinate values which represent the description of the lines of the scanning standard adopted. The polynomial function links the progressive numeration of the points which correspond to the crossing points of the scanning with the profile. Therefore, the various indicators are referred exclusiely to the graphical representation controlled by the function. It is likely that one or more of the independent variable sets can be found to be a different kind which may provide an even better outcome.

However the exploration of this possibility has been attempted only in the preliminary phase since the solution adopted here guarantees a good degree of fit and a description which in our opinion can be considered satisfactory. Our conclusions are based on the values of the statistical evaluators, and are summed both by the graphical relationships and by the evaluation of the divergence between the empirical data series and the corresponding data series obtained when the equation was solved for the same independent variables.

In addition to these parameters the computational program supports the entire equation \(b_0, b_1, b_2, b_3, \ldots b_k\) along with the relative standard deviations and variance covariance matrix. Finally, the program provides a graphic presentation of the empirical point series and of the function curve. The program elaborates the definition of the equations subsequent, in increasing distance from the second and it interrupts the computation when the polynomial with the best approximation is found (V. Pasto Focchini, 1981).

This best approximation is based on subsequent confrontations between the values of the estimated variance relative to the different equations. In the conclusions obtained for different profiles of different skulls in such strict conditions of normalization and standardization, the values of the constant \(b_0\) are very similar, as they are directly linked to the position within the adopted coordinate system.

The outline of the curve is regulated by the coefficients whose size decreases with the increase of the order up to a very small value which is necessary for a correct solution of the equation. In fact, the information included in the equation system is thus made of values completely and exactly if the coefficients are utilized without any round off or truncating: the size of the coefficients one decreases according to the better outcome of the function fit. The program includes a control flag to signal when the values of the coefficient of determination values are too small that they are equal to the machine numerical error. Since the values of the coordinates of every point of the profile are available, the program also provides some of the traditional profile calculations such as length, height, horizontal development, the three relative indices, the length of the crural and the cingular arch.

This routine can also be used for evaluations of single characteristics of limited and adequately enlarged parts of the profile. For skull sections, area, height, maximum and minimum width and the relative indices also be evaluated.

Once the equation obtained the profile can be reconstructed numerically and graphically. Linear measures and their indices can also be calculated and compared.

The main application of this procedure lies in the possibility of making comparisons which allow the definition of actual differences excluding related to form between series of fossil and modern crania. As diverse segments or sections of the skull can be analyzed, comparisons may show how different morphological distances for various parts of the crania can be made. Then transformed allometric outlines and transformed distributions of the profile (logarithmic, exponential etc.) can be obtained.

The polynomials can be constructed by calculating the mathematical formulas or by first modifying single coefficients and controlling the morphological results of these operations and then, by referring to the analysis of the allometries following these modifications.

Characteristic equations for different groups of living and fossil men and differential equations that rule morphological evolutionary changes may thus be obtained.

The compare and evaluate morphological distances two methods were used by us. In fact divergence was gauged: a) by linear regression correlation and b) by punctual measure and the relationship between the large ray curves which are present in any crania and indicates the overall similarity between the different proportions of the same cranial lines. The second aims at revealing subtle differences by evaluating the compared couples of the series (both for the ordinates and for the values obtained by the polynomial of the best approximation) and
the relative divergence expressed by the total error, total absolute error, average error, absolute average error, average square error and its square root (Gillchrist, 1970). This point by point comparison which takes into account the differences in the abscissas values for each pair of points in the two profiles relative to the same value of the ordinate (reciprocal pointual error) is applicable because of the image analyzed of positioning standardization and of the adimensional estimate. All these parameters are useful but the root of the average square error was commonly used. The usual value, the sign of the total error, and the average error compared with the value of the total absolute error and the average absolute error give the relative position of different segments of the two profiles within the given coordinate system. When the two profiles intersect, the change in position of profile segments is registered.

The divergence tests were first applied to determine how much the series of values obtained by solving the equation diverge from the empirical ones used to define the equation. We obtained an extremely reliable indication of the actual descriptive power of the equation found. This preliminary evaluation, which expresses the smoothing of the original data performed by the equation, with consequent description of the essential shape of the worked profile, is fundamental in comparing different skills. For such applications it must be remembered that the distance between the analytical description of the two images can be expressed by error values containing the real morphological differences, the methodological errors of the positioning procedure and of the dimensional normalization as well as the approximation error of the equation. The latter component will be absent when the empirical values of the images confronted used directly, but in this situation we lack the analytical tool (the equation) to investigate allometries and transformations.

RESULTS

Five equations are reported for Plecostomus transversalis (registered as FI in our catalog).

1 — Equation of total fronto-facial sagittal profile (vertebra-protrusion) (named FFE/0, a — e in fig. 2).


Coefficients: Standard deviations
B(0): .162.5301
B(1): .73
B(2): .683546893
B(3): .725104863
B(4): .69890368
B(5): .92422731
B(6): .169842137


2 — Equation of facial sagittal profile (vertebra-protrusion) (named FSE/0, a — e in fig. 2).


Coefficients: Standard deviations
B(0): .123.38075
B(1): .61156017
B(2): .012173313
B(3): .725104863
B(4): .69890368
B(5): .92422731
B(6): .169842137


3 — Equation of frontal sagittal profile (vertebra-globello) (named ASE/0, a — e in fig. 2).


Coefficients: Standard deviations
B(0): .129.943772
B(1): .744070826
B(2): .12
B(3): .094019325
B(4): .111710513
B(5): .833549065
B(6): .183729527
B(7): .191697348
B(8): .149829325


4 — Equation of the caudal sagittal profile corresponding to the frontal segment as above (vertebra-globello) (see 1 on Methods) (named PSE/0, a — d in fig. 2).


Coefficients: Standard deviations
B(0): .120.943772
B(1): .744070826
B(2): .12
B(3): .094019325
B(4): .111710513
B(5): .833549065
B(6): .183729527
B(7): .191697348
B(8): .149829325

The comparison between the series of empirical values and the series of values calculated with this equation gives the following parameters:

- Negative divergence: 48
- Positive divergence: 47
- Coincidence: 47
- Coefficient of determination in linear regression: 0.367
- Coefficient of correlation in linear regression: 0.368
- Standard error in linear regression: 0.82
- Total error: 0. Absolute total error: 104
- Quadratic total error: 128
- Average error: 0. Absolute average error: 0.44
- Quadratic average error: 0.67

Square root of the quadratic average error: 0.82

FIGURE 7. Equation VSE0. Original curve and function curve from the equation can be seen in the Left
Graph. The match of two curves of cranio-metric points depend on a small amount of measurements while the analyti-
cal evaluation of a continuous series relies on a series of much higher number of data points. For compara-
tive studies the possibility to evaluate morphological distances through the evaluation of divergence by means of the same procedure given for comparison between an original curve and correspondent function curves can be of great advantage; besides the ability to have numerical parameters of shape for clas-
ification and modelization.

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Anyone interested in the application of the procedure presented is invited to contact the authors directly for further details concerning hardware and software solutions.

Variance/covariance matrices of the given equations and lists of the coordinate values of the points of worked profiles are also available on request. Finally, the data handling section of our laboratory will apply the five equations illustrated (FPE0, FNE0, FSE0, FSEV, and FVE0) together with their matrices and point value lists, for each specimen of the following catalog where P is from standard photographs, S from actual skulls, C from casts, M from craniograms by S. Sergi and B from craniograms by M. Boule. Number after slash in the equation labels indicates angular value of shape for stereo-maps (0 for canonical lateral view).

P 1: Plenosphanostoma transvacuum (St335)
P 2: Taurin
P 3: Zingtekthra biocsi (OH1, Tobias rec.)
P 4: Homo habilis (Koobi Ford, ER1470)
P 5: Felonconsys cruris (vulcanus, Sangirian 4) (Weidenreich rec.)
P 6: Sinemostoma p Meshorensis (KoB2, Weidenreich rec.)
P 7: Broken Hill 1
P 8: Steinheim 1
P 9: La Chapelle aux Saints 1
P 10: Tchik-Tchik 1
P 11: Circeo 1
P 12: Ez-Schlauf
P 13: Amund 1
P 14: Cro-Magnon 1
P 15: Gruta dos Enfants 6 (Homo Grimaldi)
P 16: Grotte des Enfants 4
P 17: Oberkassel 1
P 18: Premolnda 3
P 19: Djebel Kafaz 5
P 20: Gamble 4
P 21: Asenser 1
P 22: Afouo-Bou-Rhommel 12
P 23: Eq-el-Ammar 2

P 24-34: H. sapiens s. modern (male-femal-adult-
infant)
P 35: H. sapiens a. Bushman, female (Crania ethica)
P 36: H. sapiens a. Aeneth, male (Crania ethica)
P 37: H. sapiens a. Neocaledonian, male (Crania ethica)
P 38: H. sapiens s. Siam, male (Crania ethica)
P 39: S.IS0:2, H. sapiens s. modern (Stereomop 0—180, male, female, adult, infant)

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Prof. V. Pesci Dellino
Institute of Zoology
and Comparative Anatomy
University of Bari
Ateneo di Bari 165/3
1-701 90 Bari, Italy.