ANTHROPOLOGIE * XXV/1 * 1987

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ANALYTICAL DESCRIPTION OF CRANIAL PROFILES BY MEANS OF KTH ORDER POLYNOMIAL EQUATIONS

PROCEDURE AND APPLICATION ON PLESIANTHROPUS TRANSVAALENSIS (STS5)

ABSTRACT — A method to describe skull profiles by means of analytical morphometrical evaluators aiming at systematic comparisons and divergence ranking is presented. The technique used is fully automized and employs a television camera interfaced with a computer having specific hardware devices and an original S. A. M. (Shape Analytical Morphometry) software package. The main routine provides both coefficients of the upper degree equation, their standard deviation and standard error, as well as variance and covariance matrices whereas accessory routines supply both standardization of positioning, normalization of the apparent size of the image to be analyzed and regulations for densitometric readings. The five upper degree polynomials for the profiles of the total i.e. fronto-facial, facial, frontal and corresponding caudal trait of the lateral left view of the Australopithecus africanus (*STS5,* Plesianthropus transvaalensis) *together with its sagittal curve are discussed in detail.*

KEY WORDS: Skull profiles — Upper degree polynomial equations — Analytical morphometry.

INTRODUCTION

One of the most important goals in morphological studies is to find descriptive methods which can reduce the observer's subjectivity and facilitate comparisons through parametrization. As stated by D'Arcy W. Thompson: "Problems of form are in the first instance mathematical problems, ... problems of growth are essentially physical problems...". With these words, in 1917, he emphasized the nature of quantitative morphology hehimself had helped to found. Fifty-seven years later, P. Dullemeijer affirmed that: "...mathematical tools are methods; they serve for the analysis, the discrimination or the generalization, but the explanation is typological, causal or historical rather than mathematical. A true mathematical idea implies a formulation of the relations of shape and form to other phenomena in terms of exactly defined mathematical parameters."

In the words of D'arcy Thompson and Dullemeijer we find the principles of rigor and essentiality such which should be applied in describing the human form. Both authors draw attention to the connection between description and structural and functional interpretations which seem to be the two keystones in the thought and works of S. Sergi. Indeed these are the basic premises needed to reach the ultimate formalizations of the description.

Some methodological aspects of the problem have been studied by Oxnard (1973, 1984), Lestrel (1974) and more recently by Ignazi et al. (1980).

As underlined by Jacobshagen (1982), the quantification of irregular organic structures is limited if approached through traditional methods, i.e. utilizing measurements taken between discreet anatomical points, since from a morphological point of view a relevant quantity of information can be lost. Classical morphometry is currently utilizing measurements and their derivate fractions with practical purposes, but none of these parameters represent the description of a shape which should be obtained by mathematical functions independently from dimension (analytical morphometry).

In our work we have examined problems of macroscopic morphology in order to carry out an anthropological study of the skull, but we feel that the procedure adopted can be applied usefully in many different fields (Pesce Delfino e Ricco 1983a, b).

The central core of the problem lies in the possibility to refer biological and anatomical shapes to curves whose analytical description is an equation.

It is well known that a description by means of an equation requires a reference system for a series of points (as large as desired) known empirically. Moreover it must provide the possibility of calculating the interpolated values and of comparing the various structures under conditions of rigorous normalization. Since the profile of a skull is merely a curve the equivalent descriptive equation defining it can be easily found. In effect a cranial section can be represented by the surface under the curve a profile which is simply the integral of the curve. Therefore a descriptive equation can be worked out as for the profile.

The first essential step in this effort consists in the definition of a perfectly controllable coordinate system allowing a standardized positioning of the profiles. These can be made adimensional by continuous focal length variation of a TV camera lens, so that all profiles considered exhibit the same apparent size. Consequently, the series of points in which the profiles are subdivided, can always be referred to the same dimensional parameters calibrated to the same values. As stated by S. Sergi (1953), "The morphological and architectural characteristics of the skull are shown by its curves or craniograms which can be obtained from the orthogonal projections once the skull has been set in a standard position. These curves can be then linked to the positions which should be chosen each time in function of morphological and functional criteria." (original text translated into English by us).

Of course in order to treat these cranial curves as a continuous series of values which can be investigated systematically, adequate analytical procedures must be defined. These call for the choice of an appropriate algorithm to determine the coefficients of the equations as well as a selection of the most efficient methods in utilizing the descriptive equations for classifications and comparisons.

The above points must be examined bearing in mind that all the procedures are completely automized. A closed television circuit is interfaced with a computer through an analog-digital converter and supplied with a control apparatus. This technical solution provides the numerical values of the coordinates of single points constituting the profile. As shall be seen later in detail, the values can be obtained by a scanning of the image which in respect to a control monitor screen proceeds from top to bottom and from left to right. As a result, the profile can be determined by a cut-off densitometric value during scanning and it can be varied by simply modifying the position of the image under analysis. Any profile of a skull can be examined either vertically, transversally, or longitudinally. In order to detect the profile, the classical craniometrical points can be used for reference, but it should be remembered that these points are only nominal indicators, since some anatomical parts may be present which instead of being situated on the profile are set on its projection. This situation is typical for the sagittal fronto-facial profile both for the superior margin of the orbits, for the lateral margin of the piriform aperture and, in some cases, it may be evident for malar bones. Hence, for example, the given term "glabella" is to be considered as referred to the most prominent point of the fronto-nasal trait. To avoid these situations, which in any case expresse real anatomical characteristics, curves obtained from craniograms by adequate profile gauges (as, for instance, the "pantagoniostato craniosteoforo" by S. Sergi) or, more simply, from suitable radiographs can be used. But televisive scanning must be performed in all cases because this is the only means to obtain a dimensionality of the different profiles required by normalization.

Both skulls or photographs obtained under standardized conditions can be used. They should be fixed on suitable stands allowing different positions and variations in the positions for standardization (for example referred to Frankfort plane or others). Obviously, this procedure can be used not only for individual profile analyses but also to map entire skulls by rotation through determined axes for given angular values. These values can be used directly for profile identification and also to define the value of the Z coordinate each time for complete analytical stereo-maps. The stands are motorized and movements are controlled by software.

Other specific applications of this technique include: a) direct evaluation of the brain outlines referred to the curves for the anterior and posterior profiles intercepted by the same scanning range: b) evaluation of endocranial measurements directly from the scanning of the skull or from the scanning of endocranial casts; c) evaluation of the thickness of soft parts by means of the comparison of the flat bone outline radiographs and of the corresponding superficial outline; d) point interpolation for damaged specimen restoration; e) practical utilization in forensic Anthropology for skull identification (Pesce Delfino V. et al. 1983).

MATERIALS

The equipment utilized included:

1. Personal computer Apple II Europlus 48 Kbytes-RAM memory and Applesoft language supplied by two disc units, graphic printer, plotter and videographic display.
2. TV camera b/w GBC Mark XIV with a zoom lens, 8—30 mm focal length.
3. A Microworks^c analog-digital converter modified to increase the scanning linearity and to vary the luminosity and contrast continuously.
4. A generator of an analog-net with a variable pass (designed and built by us) that superimposes a network of vertical and horizontal black and white lines. This network is adjustable with continuity to the television image which is shown on a control monitor (in respect to the TV image, the net is completely free of parallax).
5. A light source "spot lights" with a regulatory system to vary intensity.

6. Motorized telecontrolled stands for skulls and photographs with all the spatial movements.
7. Skulls, casts and standard photographs. For the present study a lateral left view photograph (*fig. 1*) of the *Plesianthropus transvaalensis* (Broom, Robinson a Schepers 1950) as in Du Brul (1974) was utilized.

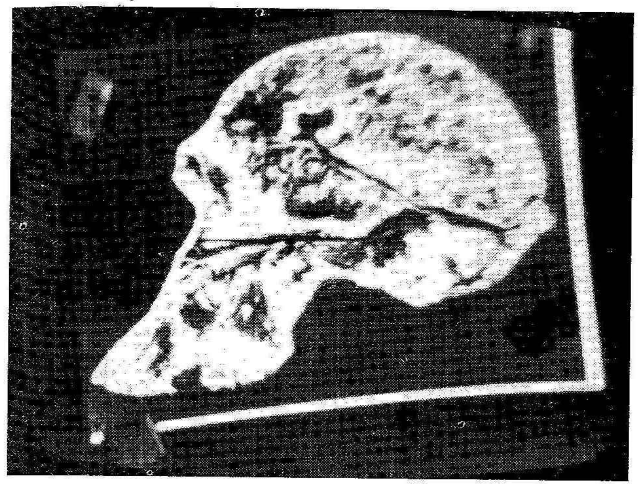


FIGURE 1. *This photo shows the skull of* Plesianthropus transvaalensis (*STS5*) (*Du Brul 1974*) *placed in the system of coordinates and visualized on the analogic monitor. A luminous reference point produced by the software is visible at the intersection between the third vertical line from the left and the seventh horizontal line from the bottom. The image is in a standard position reporting the Frankfort plane on a horizontal line while the dimensions are normalized by referring the vertex to the first scanning line, the prosthion to line 190 and the most prominent point of the posterior profile to the abscissa value of 233.*

METHODS

The software utilized was the S. A. M. (Shape Analytical Morphometry) package specifically and originally implemented in our laboratory. This package includes more than 350 routines for different items of analytical morphometry, scene analysis, image processing oriented to the study of biological structures.

The software routines used can be grouped according to their specific function as follows:

1. Routines to control the system of coordinates and the image luminosity.

The computer first generated a system of luminous points with known coordinates on a monitor where the analog TV image and the image of the variable net were visualized. To ensure repeatability of the system of coordinates the net was calibrated so that a luminous point was situated at the intersection between a given vertical with a given horizontal line (*fig. 1*). After processing a complete net was built by the software on the digitalized image present on the computer monitor to verify the congruity between the analogic and the digitalized images.

Finally, the number of lines for the scanning were determined for the treatment of a particular part of the image. For dimensional normalization different standards of number of lines were used for the scanning in different situations. A standard of 190 lines was used for 1 total fronto-facial profile (vertex-prosthion), 2. facial profile (glabella-prosthion), and for 3. sagittal profile of the vault from the glabella to the point in which the posterior profile crosses the horizontal plane where the glabella lies. A standard of 130 lines was employed for 4. frontal profile (vertex-glabella) and 5. corresponding trait of posterior profile. Therefore corresponding profiles of different specimens were subdivided into the same number of points. In respect to a previous preliminary report (Pesce Delfino et al. 1983a, b) in which a standard of 187 lines had been adopted for fronto-facial profile, the standard was now optimized to 190 lines. The coordinates of these points were expressed in arbitrary units (screen units) represented by the smallest treatable part of the digitalized image (pixel). Thus they represent the resolution absolute level (the relative one varies with the factor of the image enlargement). The parameters used were: for the abscissa interval: X_{min} 1, X_{max} 254 from left to right and for the ordinate interval: Y_{min} 1, Y_{max} 190 from top to bottom.

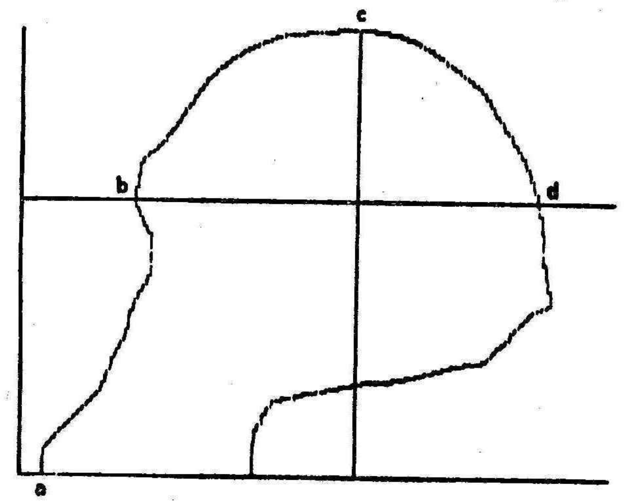


FIGURE 2. *Graphic representation of the profile obtained, through scanning. The segments considered were: a—c for equation FFE/0, a—b for equation FSE/0, b—c for equation ASE/0, c—d for equation PSE/0, b—c for equation VSE/0, segment d—e should not be considered.*

In this coordinate system the given profiles (*fig. 2*) were standardized and normalized as follows: 1. — vertex ordinate: 1, prosthion ordinate: 190, more prominent point of the occipital profile abscissa: 233; 2. — glabella ordinate: 1, prosthion ordinate: 190, prosthion abscissa: 60; 3. — glabella ordinate: 190, corresponding point on caudal profile ordinate; 1, leftmost point abscissa: 60 (in respect to others the latter profile is positioned with Frankfort plane vertical by a 90° counterclockwise rotation); 4. — vertex ordinate: 1, glabella ordinate: 130, glabella abscissa: 7; 5. — vertex ordinate: 1, ending point corresponding to glabella ordinate: 130, same point abscissa: 250.

Different profiles were namely referred to different scale factors to optimize relative enlargment of image and resolution of scanning. Nevertheless software allows to pick up as different all traits from profile of type 1. if same scale is required. For a real length of the worked profile of about 177 mm (as for fronto-facial profile of *Plesianthropus trnsvaalensis*), the average resolution was about 93 mm. This value differs in different traits since the profile orientation to the scanning can vary. In terms of incremental trend of slope (point after point delta %) we observed a range from 8 to 8. During scanning a densitometric reading classifying the image into 64 grey levels was made. The aforementioned are some of the fundamental methods of "scene analysis" (pattern recognition, image processing) based on densitometric exploration of a digitalized image with subsequent numerical and graphical elaboration. These techniques can be applied in a number of fields where automatic recognition can be useful (Hall, 1979; Rosenfeld, 1982).

In the procedure adopted the densitometric reading discriminated only between the classes of grey referred either to the background or to the image. These could be distinguished by a threshold value, and thus the value of the abscissa where the threshold was localized was registered. The luminosity, the interface contrast, and the illumination of the object under study were all controlled by intervening when necessary.

In utilizing the techniques discussed above the procedure which can be adopted is the following:

i) Selection of the skulls to be analyzed. Modern skulls are to be chosen to evaluate the differences due to individual and population variation as well as differences due to age and sex. Fossil skulls are to be worked for comparisons of phyletic relevance. Photographs from the literature showing crania in canonical norms or modern skulls or casts of fossil ones can also be used.

ii) Selection of the profile of the crania to be studied. Obviously this is possible only when the material consists of skulls or casts.

iii) Standardization of position and normalization of dimensions of the specimens. The procedure of positioning depends on whether the specimens are real skulls, casts or photographs. For real skulls or casts any position is valid, but for photographs position changes are possible only in the plane parallel to the camera. For skulls and casts, the camera should be at least one meter away to avoid central deformation. The plane of reference has to be defined in all cases.

In the present work we used the Frankfort plane (*fig. 1*). The positioning was achieved through the use of a tuned analog net by making one of its horizontal lines coincide with the Frankfort plane. Two light spots generated by the computer on the analog monitor and located on the same horizontal line were localized at the extremities of the Frankfort plane; the uppermost point of profile was located at scanning line number one and the lower point of each profile was located at the ending point of the corresponding given standard of scanning.

It must be noted that these two valus of ordinates were common to all profiles of the same traits of different images; the abscissa values of the rightmost point of the caudal (at right) profile was also common to all (equal to 233 in our coordinate system). Consequently, a standard positioning was obtained and different images were dimensionally normalized.

Actually the caudal profile alone was truly normalized because different antero-posterior lengths of different skulls caused non comparable positioning of rostral (at left) profiles. This was solved by means of the software that allowed renormalization of the rostral profiles by simply transferring the first deepest point of the naso-glabella segment to a fixed value of the abscissa (equal to 100). So all dimensional differences were eliminated and only differences of shape were considered in subsequent analytical methods.

The final step in the image adjusting phase called for the regulation of the illumination intensity so as to obtain good contrast between the image and the background. At this point it was possible to proceede with the image analysis routines.

2. Routines for the algorithm application to define the descriptive equation.

These algorithms can be of various type: skeletonizing procedures useful mostly from a graphic point of view; polynomials and analysis of the frequency spectra. In regards to the polynomials among the numerous possibilities, namely natural cubic splines, normalized polynomials, and polynomials of an upper degree, we feel that the latter are the more useful than the normalized polynomials or splines. They furnish a description by approximation, whereas the polynomial splines provide the exact matching for the series of points. The degree of interpolating function is kept very low in respect to the maximum possible degree which should correspond to the number of points minus one, in order to avoid the anomalous peaks characteristic of very high degree polynomials. This, together with the graphic control adopted method, has given good quality results though these were obtained through easy to handle equations. Moreover the smoothing effect introduced by the upper degree polynomials is particularly useful to study the curves in the skulls of which there are few samples or only one specimen available, as in the case of fossil remains. In fact the smoothing effect not only eliminates the residual methodological error procedure, it also reduces the influence of the morphologic characteristics of less importance (individual ones) present in every curve. These errors become more evident because of their gaussian distribution with respect to the function. On the contrary a periodic error distribution calls for an additional characterization of the curve. In any case we obtain an essential but characteristic curve for each worked profile.

The algorithm for the definition of the descriptive equation represented by a polynomial of K^{th} order of the type,

$$Y = b_k x^k + b_{k-1} x^{k-1} + b_{k-2} x^{k-2} + \ldots + \\ + b_3 x^3 + b_2 x^2 + b_1 x + b_0$$

is based on the least squares regression-interpolation method. If (Y_i, X_i) is the i^{th} value observed, with Y_i (dependent variable) as the abscissa value of the point

found by the scanning on the profile in correspondence to the fixed ordinate X_i (independent variable, for $i = 1, 2, 3 \ldots n$ where n is maximum ordinate value, i.e. ending line in the adopted standard of scanning), then the minimum of the following function with the variables $b_0 \ldots, b_k$ can be determined:

$$f(b_0, \ldots, b_k) = (y_i - b_0 - b_1 x_i - \ldots b_k x^k)^2$$

so as to render null all the partial derivatives with respect to the polynomial coefficients. A partial derivative equation system is thus made up of $K + 1$ equations in $K + 1$ unknown values where $K + 1$ is the number of coefficients to determine in order to estabilish the unique solution of the equation. (Cuthbert and Wood, 1980; Green and Margerison, 1979).

A fundamental problem was the selection of the independent variable values. The corresponding values of the dependent variables were represented by the abscissa values of the profile single points found by the computer program. These were based on crossing a threshold (cut-off) value, defined by the operator according to image luminosity during densitometric scanning.

According to the value of the fit goodness for the function, (high values of the coefficient of determination and low values of the standard error and of the estimated variance) for an order of the polynomial which is acceptably low in respect to the number of points, the best choice is simply the positive integer series between 1 and n. From the top to the bottom of the profiles these series can be considered as ordinate values which correspond to the progressive numeration of the lines of the scanning standard adopted.

The polynomial function links the progressive numeration of the abscissa values which correspond to the crossing points of the scanning with the profile. Therefore, the various indicators are referred exclusively to the goodness of the description controlled by the function. It is likely that one or more of the independent variable sets can be found to be of a different kind which may provide an even better outcome.

However the exploration of this possiblity has been postponed to a future research phase since the solution adopted here guarantees a goodness of fit and a description which in our opinion can be considered satisfactory. Our conclusions rest on the values of the statistical evaluators, and are supported both by the graphical restitutions and by the evaluation of the divergence between the empirical data series and the corresponding data series obtained when the equation was solved for the same independent variables.

In addition to these parameters the computational program supplies the coefficients of the equation $b_0, b_1, b_2, b_3, \ldots b_k$ along with the relative standard deviations and variance covariance matrix. Finally, the program provides a trace of the composite graph of the empirical point series and of the function curve. The program elaborates the definition of the equations sequentially, in increasing degrees, starting from the second and it interrupts the computation when the polynomial with the best approximation is found (V. Pesce Delfino et al. 1984a, b). This best approximation is based on subsequent confrontations between the values of the estimated variance relative to the different equations. In the equations obtained for same profiles of different skulls in such strict conditions of normalization and standardization, the values of the constant (b_0) are very similar and they are related to the position within the adopted coordinate system.

The outline of the curve is regulated by the coefficients; their size decreases with the increase of the order up to a very small value which is necessary for a correct solution of the equation. In fact, the information contained in the equation can be expressed completely and exactly if the coefficients are utilized without any rounding off or truncating; the size of the coefficient standard deviation also decreases according to the better outcome of the function fit. The program includes a control flag to signal when the values of the coefficient standard deviation become so small that they are equal to the machine numerical error. Since the values of the coordinates of every point of the profile are available, the program also provides some of the traditional profile calculations such as length, height, horizontal development, the three relative indices, the length of the chord and the chord/arc ratio.

This routine can also be used for evaluations of single characteristics of limited and adequately enlarged parts of the profile. For skull sections, area, height, maximum and minimum widths and the relative indices can be also evaluated.

Once the equation is obtained the profile can be reconstructed numerically and graphically. Linear measures and their indices can also be recalculated and compared.

The main application of this procedure lies in the possibility of making comparisons which allow the definition of actual morphological distances exclusively related to form between series of fossil and modern crania. As diverse segments or sections of the skull can be analyzed, comparisons which show the different morphological distances for various parts of the crania can be made. Then transformed allometric outlines and transformed restitutions of the profile (logarithmic, exponential etc.) can be also obtained. The polynomials can be constructed by calculating the arithmetical, harmonical or geometrical average or by first modifying single coefficients and controlling the morphological results of these operations and then by referring to the analysis of the allometries following these modifications.

Characteristic equations for different groups of living and fossil men and differential equations that rule morphological evolutionary changes may thus be obtained.

To compare and evaluate morphological distances two methods were used by us. In fact divergence was gauged: a) by linear regression correlations and b) by punctual reciprocal error. The first procedure measures the relationship between the large ray curves which are present in any crania and indicates the overall similitary or the overall differences between the outlines. The second aims at revealing subtle differences by evaluating the compared couples of the series (both for the empirical data and for the values obtained by the polynomial of the best approximation) and the relative divergence expressed by the total error, total absolute error, average error, absolute average error, average square error and its square root (Gilchrist, 1976). This point by point comparison which takes into account the differences in the abscissa values for each pair of points in the two profiles relative to the same value of the ordinate (reciprocal punctual error) is applicable because of the image analyzed of positioning standardization and of the adimensional outcome. All these parameters are useful but the root of the average square error was commonly used. The numerical value, the sign of the total error, and the average error compared with the value of the total absolute error and the absolute average error give the relative position of different segments of the two profiles within the given coordinate system.

When the two profiles intersect, the change in position of profile segments is registered.

The divergence tests were first applied to determine how much the series of values obtained by solving the equation differ from the empirical ones used to define the equation. We obtained an extremely reliable indication of the actual descriptive power of the equation found. This preliminary evaluation, which expresses the smoothing of the original data performed by the equation, with consequent description of the essential shape of the worked profile, is fundamental in comparing different skulls. For such applications it must be remembered that the distance between the analytical description of the two images will be expressed by error values containing the real morphological differences, the methodological errors of the positioning procedure and of the dimensional normalization as well as the approximation error of the equation. The latter component will be absent when the empirical values of the images confronted are used directly, but in this situation we lack the analytical tool (the equation) to investigate allometries and transformations.

RESULTS

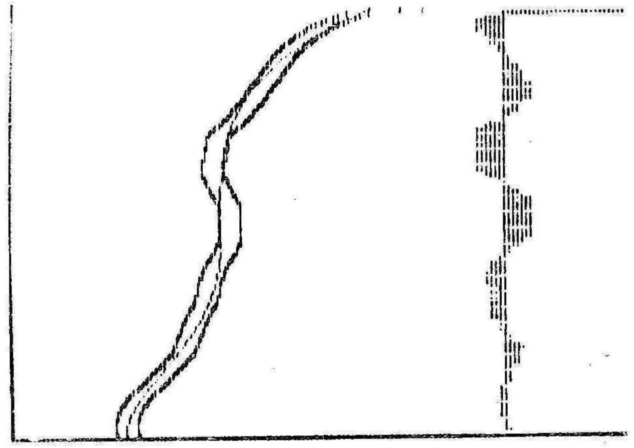
Five equations are reported for *Plesianthropus transvaalensis* (registered as P1 in our catalog).

1 — *Equation of total fronto-facial sagittal profile (vertex-prosthion) (named FFE/0, a—c in fig. 2).*

Degree: 6th Square residuals: 1611.58. Variance: 8.80. Coefficient of determination: .98.

Coefficients	Standard deviations
B(0):166.423301	1.61
B(1):—2.56557262	.23
B(2):.0395940893	.01
B(3):—4.02994953E-04	2.05E-04
B(4):3.3289879E-06	1.94E-06
B(5):—1.76395225E-08	8.85E-09
B(6):3.69165188E-11	1.53E-11

The comparison between the series of empirical values and the series of values calculated with this equation gives the following parameters: Negative divergences: 92. Positive divergences: 81. Coincidences: 17. Coefficient of determination in linear regression: .985. Coefficient of correlation in linear regression: .992. Standard error in linear regression: 2.937. Total error: —2. Absolute total error: 446. Quadratic total error: 1646. Average error: —.01. Absolute average error: 2.34. Quadratic average error: 8.66. Square root of the quadratic average error: 2.94.



FIGURES 3. *Original curve and function curve from the equation can be seen in the left graph. If they were superimposed it would be difficult to distinguish them, the two external lines, which are equal, represent the series of original values while the central one represents the interpolating function curve. The match of two curves with graphical exaggeration of reciprocal error can be noticed in the right graph (on the right positive divergences, on the left negative divergences of reconstructed curve in respect to original curve).*

2 — *Equation of facial sagittal profile (glabella-prosthion) (named FSE/0, a—b in fig. 2)*

Degree: 6th. Square residuals: 192.63. Variance: 1.05. Coefficient of determination: .99.

Coefficients	Standard deviations
B(0):123.980793	.55
B(1):.611556117	.08
B(2):.0107013133	3.62E-03
B(3):-7.25193192E-04	7.09E-05
B(4):9.8863898E-06	6.74E-07
B(5):-5.49237941E-08	3.06E-09
B(6)-1.08942137E-10	5.32E-12

The comparison between the series of empirical values and the series of values calculated with this equation gives the following parameters: Negative divergences: 60. Positive divergences: 59. Coincidences: 71. Coefficient of determination in linear regression: .998. Coefficient of correlation in linear regression: .999. Standard error in linear regression: 1.045. Total error: -4. Absolute total error: 148. Quadratic total error: 206. Average error: -.02. Absolute average error: .77. Quadratic average error: 1.08. Square root of the quadratic average error: 1.04.

3 — *Equation of frontal sagittal profile (vertex-glabella) (named ASE/0, b—c in fig. 2)*

Degree: 8th. Square residuals: 842.44. Variance: 6.96. Coefficient of determination: .99.

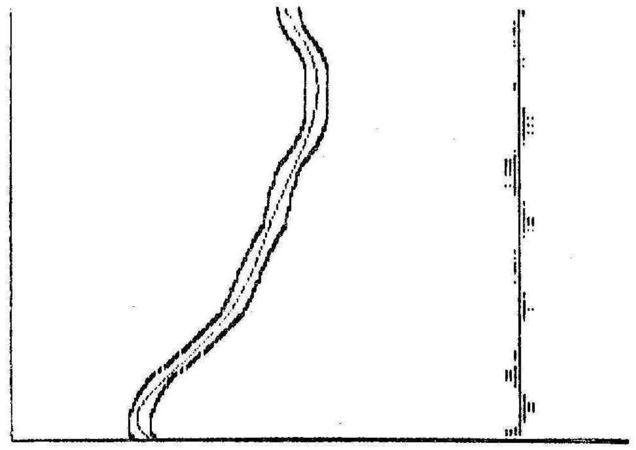


FIGURE 4. *Equation FSE/0. Original curve and function curve from the equation can be seen in the left graph. The match of two curves with graphical exaggeration of reciprocal error can be noticed in the right graph.*

Coefficients	Standard deviations
B(0):174.922092	1.76
B(1):-9.29756243	.27
B(2):.341439723	<10E-14
B(3):-4.22398427E-03	<10E-14
B(4):-8.29890306E-05	<10E-14
B(5):3.26572307E-06	<10E-14
B(6):-4.11563582E-08	<10E-14
B(7):2.35212393E-10	<10E-14
B(8):-5.14175581E-13	<10E-14

The comparison between the series of empirical values and the series of values calculated with this equation gives the following parameters: Negative divergences: 45. Positive divergences: 49. Coincidences: 36. Coefficient of determination in linear regression: .995. Coefficient of correlation in linear regression: .997. Standard error in linear regression: 2.51. Total error: -5. Absolute total error: 185. Quadratic total error: 817. Average error: -.03. Absolute average error: 1.42. Quadratic average error: 6.28. Square root of the quadratic average error: 2.50.

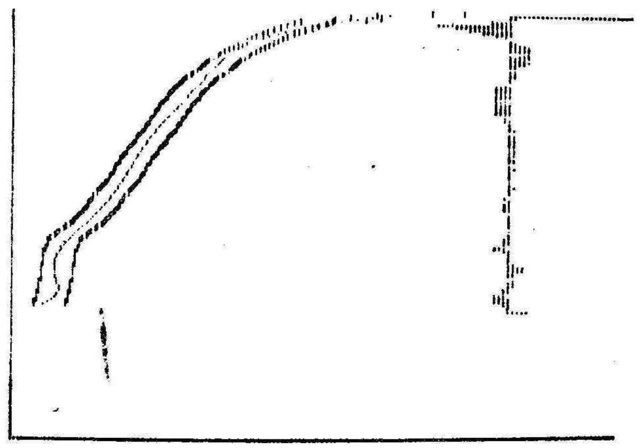


FIGURE 5. *Equation ASE/0. Original curve and function curve from the equation can be seen in the left graph. The match of two curves with graphical exaggeration of reciprocal error can be noticed in the right graph.*

4 — *Equation of the caudal sagittal profile corresponding to the frontal segment as above (vertex-glabella) (see 1 in Methods) (named PSE/0, c—d in fig. 2).*

Degree: 8th. Square residuals: 171.36. Variance: 1.41. Coefficient of determination: .99

Coefficients	Standard deviations
B(0):120.943772	.79
B(1):3.71470826	.12
B(2):-.0584919325	<10E-14
B(3):-1.11719613E-03	<10E-14
B(4):8.32634008E-05	<10E-14
B(5):-1.80720572E-06	<10E-14
B(6):1.91917349E-08	<10E-14
B(7):-1.0103171E-10	<10E-14
B(8):2.10498284E-13	<10E-14

The comparison between the series of empirical values and the series of values calculated with this equation gives the following parameters: Negative divergences: 49. Positive divergences: 45. Coincidences: 36. Coefficient of determination in linear regression: .998. Coefficient of correlation in linear regression: .999. Standard error in linear regression: 1.18. Total error: -3. Absolute total error: 115. Quadratic total error: 181. Average error: -.02. Absolute average error: .88. Quadratic average error: 1.39. Square root of the quadratic average error: 1.17.

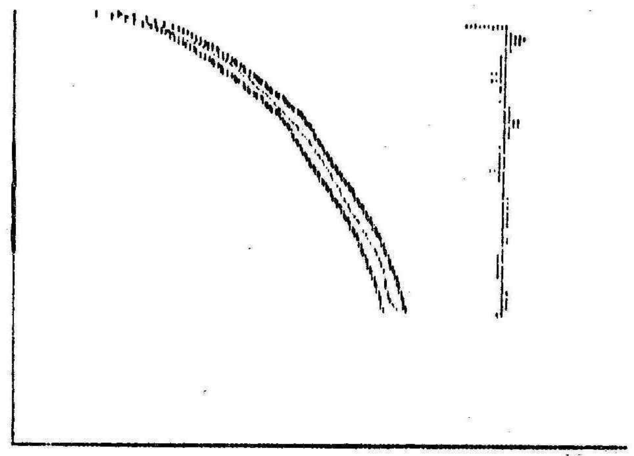


FIGURE 6. *Equation PSE/0. The original curve and function curve from the equation can be seen in the left graph. The match of two curves with graphical exaggeration of reciprocal error can be noticed in the right graph.*

5 — *Equation of the sagittal profile of the vault from the glabella to the ending point as above (see 1 in methods) (named VSE/0, b—d in fig. 2).*

Degree: 6th. Square residuals: 110.96. Variance: .60. Coefficient of determination: .99.

Coefficients	Standard deviations
B(0):116.231688	.42
B(1):-1.75242817	.06
B(2):.0167544821	2.75E-3
B(3):3.8788855E-05	5.38E-5
B(4):-1.8899851E-06	5.11E-7
B(5):1.28535749E-08	2.32E-9
B(6):-2.6889509E-11	4.03E-12

The comparison between the series of empirical values and the series of values calculated with this equation gives the following parameters:
Negative divergences: 49. Positive divergences: 47. Coincidences: 94. Coefficient of determination in linear regression: .997. Coefficient of correlation in linear regression: .998. Standard error in linear regression: .82. Total error: 0. Absolute total error: 104. Quadratic total error: 128. Average error: 0. Absolute average error: .54. Quadratic average error: .67. Square root of the quadratic average error: .82

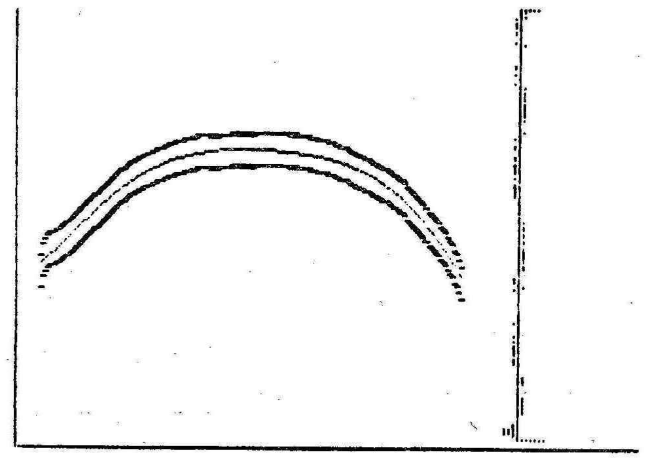


FIGURE 7. *Equation VSE/0. Original curve and function curve from the equation can be seen in the left graph. The match of two curves with graphical exaggeration of reciprocal error can be noticed in the right graph.*

From a purely descriptive point of view (i.e. for individual descriptions) the fronto-facial profile may be represented by a couple of separate equations rather than by a single equation, since the divergence between the empirical values and values obtained by the equations is much smaller in the first case than in the second while the resolution is higher. Yet, the single equation is strongly descriptive and strictly related to the skull in question ("characteristic"), hence it can be used for any comparison or grouping. In consideration of the decreasing value of the standard error, the equations reported can be listed as follows: 1-FFE/0, 2-ASE/0, 3-PSE/0, 4-FSE/0,5 -VSE/0. This indicates the quality of the fitness obtainable through them. It can also be observed that the least elevated maximum degree (6^{th}) was found for the following equations: 1-FFE/0, 2-FSE/0, 3-VSE/0 while equations ASE/0 and PSE/0 reached 8^{th} degree but the extremely low standard deviation value for coefficients higher than second leads us to refer such curves to ordinary ones. Therefore found function curve, always characteristic, may be also referred to basic types. The first three equations refer to curves with a more complex trend than the last two: equations (FFE/0 and FSE/0) due to their irregular fronto-facial outline and equation VSE/0 because of the total inversion of trend of the curve between the upper and lower half. The curves represented by equations ASE/0 and PSE/0 are of a simpler type and the standard error value of equation ASE/0 must be due to the fact that it also includes the supraorbital ridge. The standard deviation value of the constant B(0) of the different equations, corresponding to a vertical straight line whose localization depends from positioning standard, represents also a useful indicator for the evaluation of the symmetry of the upper and lower half of each curve.

Our method should be considered both for descriptive and systemtic problems and for comparisons. It is notable that descriptions of skulls based on discrete measures between craniometric points depend on a small amount of measurements while the analytical evaluation of a continuous series relies on a series of much higher number of data points. For comparative studies the possibility to evaluate morphological distances through the evaluation of divergence by means of the same procedure given for comparison beetwen an original curve and correspondent function curve can be of great advantage; besides the ability to have numerical parameters of shape for classification and modelization.

This research was carried out through funds from the Ministero della Pubblica Istruzione, 40% grants. Anyone interested in the application of the procedure presented is invited to contact the authors directly for further details concerning hardware and software solutions. Variance/covariance matrices of the given equations and lists of the coordinate values of the points of worked profiles are also available on request. Finally, the data handling section of our laboratory will supply the five equations illustrated (FFE/n, FSE/n, ASE/n, PSE/n, VSE/n) together with their matrices and point value lists, for each specimen of the following catalog where P is from standard photographs, S from actual skulls, C from casts, M from craniograms by S. Sergi and B from craniograms by M. Boule. Number after slash in the equation labels indicates angular rotation values for stereo-maps (0 for canonical lateral norm):

P 1: *Plesianthropus transvaalensis* (STS5)
P 2: Taung 1
P 3: *Zinjanthropus boisei* (OH5, Tobias rec.)
P 4: *Homo habilis* (Koobi Fora, ER1470)
P 5: *Pithecanthropus erectus* (robustus, Sangiran 4) (Weidenreich rec.)
P 6: *Sinanthropus pekinensis* (CKn.B2) (Weidenreich rest.)
P 7: Broken Hill 1
P 8: Steinheim 1
P 9: La Chapelle aux Saints 1
P 10: Teshik-Tash 1
P 11: Circeo 1
P 12: Es-Skhul 5
P 13: Amud 1
P 14: Cro-Magnon 1
P 15: Grotte des Enfants 6 (Homo Grimaldi)
P 16: Grotte des Enfants 4
P 17: Oberkassel 1
P 18: Predmosti 3
P 19: Djebel Kafzeh 5
P 20: Gamble 4
P 21: Asselar 1
P 22: Afalou-Bou-Rhummel 12
P 23: Erq-el-Ahmar 2
P 24- P34: *H. sapiens s.*, modern (male-female-adult-infant)
P 35: *H. sapiens s.* Bushman, female (Crania ethnica)
P 36: *H. sapiens s.* Aleut, male (Crania ethnica)
P 37: *H. sapiens s.* Neocaledonian, male (Crania ethnica)
P 38: *H. sapiens s.* Siam, male (Crania ethnica)
S 1-S10: *H. sapiens s.*, modern (Stereomaps 0—180°, male, female, adult, infant)
C 1: *Australopithecus prometheus* (Stereomap 0—180°; rec. from Makapansgat calvaria MLD1. Cast in Museum of Istituto di Anthropologia, Padua).
C 2: Taung 1 (Stereomap, incomplete; cast in Museum of Istituto di Antropologia, Padua)
C 3: La Chapelle aux Saints 1 (Stereomap 0—180°; cast in Museum of Istituto di Anthropologia, Padua)
M 1: Circeo 1 (sagittal curve on glabella-opistocranion plane)
M 2: Saccopastore 1 (sagittal curve on glabella-opistocranion plane)
M 3: La Ferrassie (sagittal curve on glabella-opistocranion plane)
M 4: La Chapelle aux Saints 1 (sagittal curve on glabella-opistocranion plane)
M 5: Circeo 1 (sagittal curve on orbital-auricular plane)
B 1: La Chapelle aux Saints 1 (sagittal curve on glabella-inion plane)
B 2: Neanderthal (sagittal curve on glabella-inion plane)
B 3: Spy 1 (sagittal curve on glabella-inion plane)
B 4: Spy 2 (sagittal curve on glabella-inion plane)

ACKNOWLEDGEMENTS

The authors would like to thank prof. M. Di Bacco for valuable comments, prof. V. Mitolo for help in the revision of the manuscript and Prof. G. Alciati for precious suggestions and permission to handle casts of Museum of Istituto di Antropologia in Padua.

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