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SOME CONSIDERATIONS ON THE USE OF THE QUOTIENT OF SAMPLE MEANS AS A MEASURE OF SEXUAL DIMORPHISM

ABSTRACT: Assuming we are sampling from independent normal populations, the quotient of sample means is probably the most popular measure of sexual dimorphism found in the anthropological literature. However, it is used as a descriptive (vs. inferential) sample function in spite of its evident relationship with the difference of two sample means. Here we discuss some aspects of the use of this quotient, like its reliance on the independence assumption between sexes, and show a simple procedure that involves applying to the quotient of sample means the inferences that can be drawn from the difference of sample means.

KEY WORDS: Sexual dimorphism measure – Sample means quotient – Independence – Student's t distribution

THE ANTHROPOLOGICAL BACKGROUND

Smith (1999), in his work on measures of sexual dimorphism, alludes to the fact that, in animal species, there are differences between males and females in relation to the mean of the body size. Furthermore, Smith points out that Darwin was interested in exploring some selective processes related to sex dimorphism and such dimorphism has since motivated much research within evolutionary theory.

Sexual dimorphism, by its very own nature, is a component to be taken into consideration on analyzing the morphological variability of a population composed of two sexes. In what fossil samples are concerned, two types of variability can be observed. On one hand, between species variability and, on the other, within species variability, sexual dimorphism constituting one of the latter's components. A particularly interesting issue in Paleoanthropology involves differentiating these two types of variability. Plavcan (2002), following the work of Lockwood (1999), has explored some sexual dimorphism patterns across species.

Alexander *et al.* (1979) and Clutton-Brock (1985), among others, hypothesize that sexual dimorphism in higher Primates is correlated to their behavior and social organization. In this regard, Fleagle *et al.* (1980) and Kay (1982) have suggested that the behavior of extinct species can be deduced by studying the existent sexual dimorphism in fossil data.

Assessment of sex ratios in human fossil remains provides information, according to Speth (1983) and Davis (1987), on population biology. Evolutionary trends and population dynamics can be disclosed by assessment of sex ratios (Dong 1997), and Klein, Cruz-Uribe (1983) have suggested that body size estimates can be affected by the sex ratio calculated from the sample studied. Lee (2001) believes that sexual dimorphism, within higher vertebrates, is correlated to sex ratio. This type of dimorphism is, according to the latter author, the main component in the variability of a population.

In summary, sexual dimorphism appears to be, from a biological perspective, conceptually clear and, as a component of population variability, of particular significance. However, there is no consensus as to which method should be used to measure this dimorphism. Indeed, numerous indices or measures have been proposed for evaluating sexual dimorphism but the biological community does not consider them to be reliable in equal measure. Here we focus on the index, named quotient of sample means that is more frequently used in sexual dimorphism studies, at least in the anthropological field. A more general discussion on indices of sexual dimorphism can be found elsewhere (Ipiña, Durand 2010).

THE LOG-TRANSFORMED QUOTIENT

The quotient of sample means (Q) which is defined as,

$$Q = \frac{\bar{X}_1}{\bar{X}_2},$$

where $\bar{X}_i, i = 1, 2$ (male, female), are the sample means corresponding to the two sexes subject to analysis, is probably the most popular measure of sexual dimorphism used by anthropologists. The work of Lovich and Gibbons (1992) is an example where the interested reader can see other forms of this quotient, with $\log(Q)$ being the most prevalent.

This latter log-transformed form deserves, in our opinion, some comment. On the one hand, the transformation seems to solve some problems related to the bias and kurtosis of the empirical (vs. analytical) distribution of the quotient (see, e.g., Ranta *et al.* 1994). On the other hand, it should to be kept in mind that, by applying this transformation, the distributional properties of the random variables involved change. Thus, let us assume that we are sampling from independent normal populations so that the quotient Q is constructed with two independent normally distributed sample means. By considering these constraints, the interested reader can see in Ipiña (2002) the analytical expression of the Q density (see an example in *Figure 1*). Now, let us assume that $\log(Q)$ is normally distributed. This entails that Q is lognormally distributed and, as can be seen in *Figure 1*, the lognormal density and the density derived by Ipiña are different.

Accordingly, if we are sampling from independent normal populations, then Q is not lognormally distributed which is the same as saying that $\log(Q)$ is not normally distributed. If, on the contrary, we suppose that $\log(Q)$ is normally distributed then either we are not sampling from normal populations or these populations are not independent. With $\sim^c$ standing for "not distributed as" and $\perp^c$ standing for "not independent from", we have,

$$\left\{\left(\bar{X}_1, \bar{X}_2 \sim \text{normals}\right) \cap \left(\bar{X}_1 \perp \bar{X}_2\right)\right\} \Rightarrow \\ \Rightarrow Q \sim^c \text{lognormal} \Rightarrow \log(Q) \sim^c \text{normal},$$

hence,

$$\log(Q) \sim \text{normal} \Rightarrow Q \sim \text{lognormal} \Rightarrow \\ \Rightarrow \left\{\left(\bar{X}_1, \bar{X}_2 \sim \text{normals}\right) \cap \left(\bar{X}_1 \perp \bar{X}_2\right)\right\}^c,$$

that is to say,

$$\left\{\left(\bar{X}_1, \bar{X}_2 \sim^c \text{normals}\right) \cup \left(\bar{X}_1 \perp^c \bar{X}_2\right)\right\}.$$

In short, it does not seem tenable to assume that $\log(Q)$ is normally distributed and, at the same time, to suppose that we are dealing with independent and normally distributed populations.

ARE THE SEXES INDEPENDENT POPULATIONS?

When analyzing sexual dimorphism, it is customary, in the (paleo-) anthropological field, to assume independent normal populations. We have discussed elsewhere (Ipiña, Durand 2010), the consequences of assuming independent populations, as the following example shows.

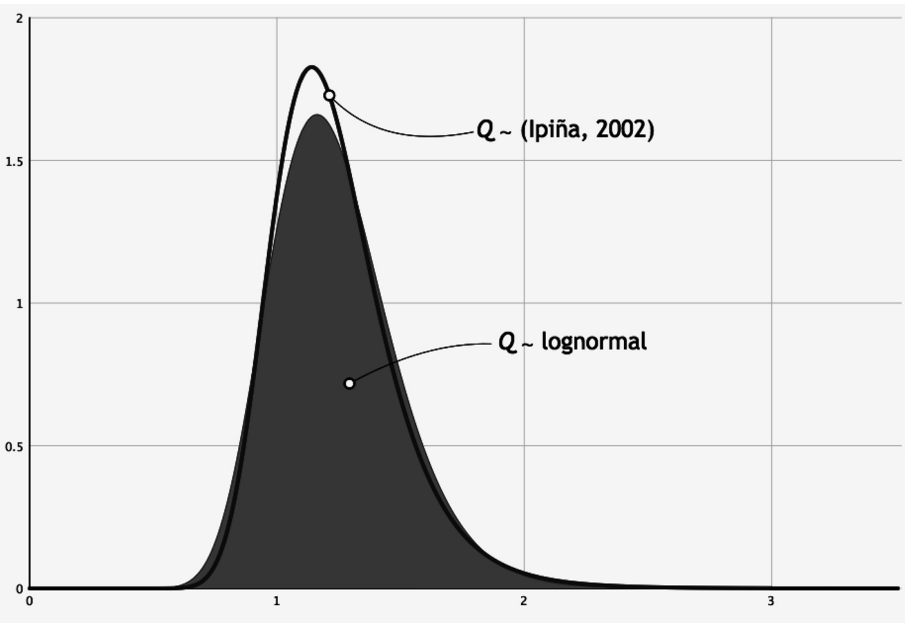

FIGURE 1. An example of the comparison between the densities of the quotient of two sample means which are independent normal populations (Ipiña, 2002) and of the lognormal distribution.

Example 1. – In probability theory, it is well-known that two events S_1 and S_2 are independent if,

$$P(S_1|S_2) = P(S_1) \text{ and } P(S_2|S_1) = P(S_2),$$

or, which is the same,

$$P(S_1 \cap S_2) = P(S_1)P(S_2).$$

As an immediate consequence, it is also known that two distribution functions F_x and F_y are independent if,

$$F(x,y) = F_x(x)F_y(y),$$

F being the joint bivariate distribution function of the random variables X and Y.

Let us assume that the two random variables "female heights of a human population" (X) and "male heights of the same population" (Y) are analyzed. If these sets of women and men are considered to be independent, one can easily compute the probability of simultaneously observing the events females whose height is strictly less than 1.51 m. and males whose height is strictly less than 1.51 m.,

$$P[(X < 1.51) \cap (Y < 1.51)] =$$
$$= F(1.51, 1.51) = F_x(1.51)F_y(1.51).$$

This is the same as stating that,

$$F_y(1.51) = F_{y|x}(1.51|1.51),$$

$F_{y|x}$ being the conditional distribution function of male heights in respect to female heights.

This means that the above mentioned male heights are, according to the hypothesis of independence between sexes, independent of the fact that women such as, e.g., their mothers or grandmothers are, or have been, strictly less than 1.51 m. tall.

On the other hand, let S be the event "to belong to the M sex" so, its complement S^c will then be "to belong to the F sex" (assuming there are two sexes, named M and F, in the population). Are two complementary events, others than the sure and impossible events, independent? That is to say, is it true that,

$$P(S \cap S^c) = P(\varnothing) = 0 = P(S)P(S^c)?$$

As is well known, on the contrary, if, e.g., $P(S) = 0.48$, then,

$$P(S^c) = 0.52 \Rightarrow P(S)P(S^c) \neq 0.$$

IS A VALUE OF THE QUOTIENT OF SAMPLE MEANS SIGNIFICANTLY DIFFERENT FROM 1?

A problem with Q is that, assuming independent normal populations, its density function is not an explicit one (Ipiña 2002) so, trying to make inferences with this random variable becomes a hard task. This is probably the reason why, to the authors's knowledge, the frequently used p-value that accompanies a value of any sample function with inferential support, is not presented alongside a sample means quotient value.

Our aim is to propose a procedure that remedies this state of affairs. It consists in that, assuming that independent normal populations are involved, inferences drawn from the difference of sample means will equally apply to the quotient of sample means. When trying to make inferences, we are interested in testing whether a specific value of the quotient is significantly different from 1, or not. In terms of sexual dimorphism this is the same as analyzing whether or not such dimorphism exists.

Assuming that the quotient of two mathematical expectations is the parametric counterpart of the quotient of two sample means, it is plain that,

$$\frac{\mu_1}{\mu_2} = 1 \Leftrightarrow \mu_1 - \mu_2 = 0.$$

Thus, by performing a simple t-test to show, with a given confidence, that two sample means differ significantly, we have a way for the user to test whether or not a value of the quotient of two sample means differs significantly from the unity (examples of the application of t-tests to sexual dimorphism studies can be seen in Greene 1989). As widely known, this is done by defining a sample function that has two algebraic forms, depending on whether the sample variances are significantly different, or not. If sample variances do not differ significantly, the sample function is, with $W = \bar{X}_1 - \bar{X}_2$,

$$T = \frac{W - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_{(n_1+n_2-2},$$

where the null hypothesis $H_0 : \mu_1 - \mu_2 = 0$ must be included, given our aim. In the case that sample variances differ significantly, as also widely known, the sample function is defined and approximately distributed as,

$$T' = \frac{W - H_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t_{(\omega},$$

$$\omega = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}},$$

Example 2. – Suppose we have two independent random samples with,

$$\bar{x}_1 = 76.7,\ \bar{x}_2 = 67.6,$$
$$s_1^2 = 2.54,\ s_2^2 = 3.05,$$
$$n_1 = 6,\ n_2 = 4,$$

so $q = 1.1346$ and $w = 9.1$. With this w difference we have $t = 8.52139$ which p-value is 0.0000552 considering equal variances (F test p-value = 0.7989). As a consequence, the value $q = 1.1346$ of the quotient of sample means differs significantly from 1.

Now, let us suppose that,

$$\bar{x}_1 = 76.7,\ \bar{x}_2 = 67.6,$$
$$s_1^2 = 0.1857,\ s_2^2 = 35.8247,$$
$$n_1 = 6,\ n_2 = 4,$$

so, as before, $q = 1.1346$ and $w = 9.1$. Thus, we have $t' = 3.00854$ which p-value is 0.05676 considering that sample variances now differ significantly (F test p-value = 0.00003). Hence, in this new case, the value $q = 1.1346$ of the quotient of sample means does not differ significantly from 1 at a significance level 0.05 or less.

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